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Tests for nonergodicity of denumerable continuous time Markov processes[☆]

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Abstract

We provide nonergodicity criteria for denumerable continuous time Markov processes in terms of test functions that satisfy Kaplan's condition, which resolves an open problem given by [B.D. Choi, B. Kim, nonergodicity criteria for denumerable continuous time Markov processes, and Operations Research Letters 32 (2004) 575–580]. We give two examples where the nonergodicity criteria are used.

Keywords: Nonergodicity; Kaplan's condition; Markov process; Queueing theory; Retrial queue

1. Introduction

Ergodicity and nonergodicity criteria for denumerable Markov processes have been studied for several decades. Foster [1] and Pakes [2] gave ergodicity criteria for denumerable discrete time Markov processes (DTMPs). Reuter [3] and Tweedie [4,5] studied ergodicity criteria for denumerable continuous time Markov processes (CTMPs). The paper of Anderson [6] is also a good article on ergodicity criteria for denumerable CTMPs. Kaplan [7] and Sennott et al. [8, 9] gave nonergodicity criteria for denumerable DTMPs. Several ergodicity and nonergodicity criteria for denumerable DTMPs are found in Fayolle, Malyshev and Menshikov [10]. Choi and Kim [11] studied nonergodicity criteria for denumerable CTMPs.

To use ergodicity and nonergodicity criteria with test functions is almost standard in the stability analysis of queueing models in recent works. For examples, Falin [12] and He, Li and Zhao [13] studied the stability of retrial queueing systems by using ergodicity and nonergodicity criteria with test functions. For further examples, see Diamond and Alfa [14], Falin [15] and Hanschke [16].

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In this paper, we provide nonergodicity criteria for denumerable CTMPs in terms of test functions satisfying Kaplan's condition. Kaplan's condition was first introduced by Kaplan [7] for DTMPs, and it was restated by Sennott [9] in a more general form as follows: Consider a DTMP with state space E and one-step transition probabilities $p_{ij}, i, j \in E$. A function $f : E \to [0, \infty)$ is said to satisfy Kaplan's condition if

$$\inf_{c \le z < 1, i \in E} \frac{\sum_{j \in E} p_{ij} z^{f(j)} - z^{f(i)}}{z - 1} > -\infty \quad \text{for some } c \in [0, 1). \tag{1}$$

Kaplan [7] gave a nonergodicity criterion for DTMPs and Sennott [9] extended Kaplan's result and gave the following criterion:

A DTMP *is not ergodic if there exists a function* $f : E \to [0, \infty)$ *that satisfies*

Kaplan's condition (1);
$\sum_{j \in E} p_{ij} f(j) < \infty$ *for all* $i \in E$;
$\sum_{j \in E} p_{ij} f(j) \ge f(i)$ *for all* $i \in E$;
$\sum_{j \in E} p_{ij} f(j) > f(i)$ *for some* $i \in E$.

In fact, the condition "$\sum_{j \in E} p_{ij} f(j) < \infty$ for all $i \in E$" is redundant as we see in Section 2. Sennott [9] also proved that Kaplan's condition is weaker than

$$\sup_{i \in E} \sum_{j \in E} p_{ij} (f(i) - f(j))^+ < \infty, \tag{2}$$

where $(f(i) - f(j))^+ \equiv \max\{f(i) - f(j), 0\}$. Because the condition (2) can be checked more easily than (1) in general, it is suggested that one check (2) prior to (1) in practical applications. As a continuous time analogue of the Sennott's result, Choi and Kim [11] gave the following nonergodicity criterion for CTMPs:

A regular CTMP *with denumerable state space* E *and* Q*-matrix* $(q_{ij})_{i,j \in E}$ *is not ergodic if there exists a nonconstant function* $f : E \to [0, \infty)$ *that satisfies*

$$\sup_{i \in E} \sum_{j \in E} q_{ij} (f(i) - f(j))^+ < \infty; \tag{3}$$

$$\sum_{j \in E} q_{ij} f(j) \ge 0 \quad \text{for all } i \in E.$$

Choi and Kim [11] also introduced a continuous time version of Kaplan's condition (1) as follows:

$$\inf_{c \le z < 1, i \in E} \frac{\sum_{j \in E} q_{ij} z^{f(j)}}{z - 1} > -\infty \quad \text{for some } c \in [0, 1). \tag{4}$$

The condition (4) is also called Kaplan's condition when a CTMP is considered. Choi and Kim [11] proved that Kaplan's condition (4) for CTMPs is weaker than (3), which is a continuous time analogue of Sennott's result that Kaplan's condition (1) for DTMPs is weaker than (2). However, it is not known whether the condition (3) can be replaced with Kaplan's condition (4) for their nonergodicity criteria. This paper gives a positive answer to the problem.

The remainder of the paper is organized as follows: In Section 2, we provide nonergodicity criteria for denumerable CTMPs in terms of test functions that satisfy Kaplan's condition, which resolves an open problem given by Choi and Kim [11]. In Section 3, we give two examples where the nonergodicity criteria are used. In the Appendix, we summarize some criteria for the ergodicity and nonergodicity of denumerable Markov processes found in the literature.

2. Nonergodicity criteria for denumerable CTMPs

Nonergodicity criteria for CTMPs are given in the following theorem.