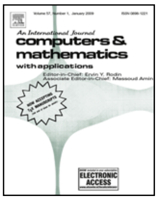
Computers and Mathematics with Applications 61 (2011) 1912–1916

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa



Nonlinear analysis and control of the uncertain micro-electro-mechanical system by using a fuzzy sliding mode control design

Her-Terng Yau [a,*], Cheng-Chi Wang [b], Chin-Tsung Hsieh [a], Ching-Chang Cho [c]

[a] *Department of Electrical Engineering, National Chin-Yi University of Technology, Taichung, Taiwan*
[b] *Department of Mechanical Engineering, Far East University, Hsin-Shih, Tainan, Taiwan*
[c] *Department of Mechanical Engineering, National Cheng-Kung University, Tainan, Taiwan*

ARTICLE INFO

Keywords:
MEMS
Chaos
Fuzzy sliding mode control (FSMC)
Sliding surface

ABSTRACT

This study analyzes the chaotic behavior of a micromechanical resonator with electrostatic forces on both sides and investigates the control of chaos. A phase portrait, maximum Lyapunov exponent and bifurcation diagram are used to find the chaotic dynamics of this micro-electro-mechanical system (MEMS). To suppress chaotic motion, a robust fuzzy sliding mode controller (FSMC) is designed to turn the chaotic motion into a periodic motion even when the MEMS has system uncertainties.

1. Introduction

Nonlinearities exist ubiquitously in micro-electro-mechanical systems (MEMS). Examples include nonlinear springs and damping mechanisms [1], nonlinear resistive, inductive and capacitive circuit elements [2] and nonlinear surface, fluid, electric and magnetic forces [3]. Many researches have been conducted on various nonlinear dynamic phenomena, including bending of the frequency response curve and the jump phenomenon in MEMS resonators [4]. Nonlinearities may also cause chaotic behavior [5]. Modeling [6] has been used to predict the existence of chaotic motion in electrostatic MEMS. In one study [7], the chaotic motion of MEMS resonant systems close to the specific resonant separatrix was investigated under the corresponding resonant condition. An optimal linear feedback control strategy has been adopted [8] to reduce the chaotic motion of the system proposed in the former study [7] to a stable orbit. In a later investigation [9], the chaotic behavior of a micro-electro-mechanical oscillator was modeled by a version of the Mathieu equation and was studied both numerically and experimentally. Chaotic motion of a micro-electro-mechanical cantilever beam under both open and close loop control has also been reported [10].

This study develops a fuzzy sliding mode control (FSMC) scheme [11–13] that is designed to control chaos in a MEMS with system uncertainties. Firstly, the switching surface that is required to achieve chaos control is specified, and then a switching control law based on fuzzy linguistic rules is developed to generate a suitable chatter-free control signal for driving the error dynamic system such that the error state trajectories converge asymptotically to zero.

2. System description

Fig. 1 presents the electrostatically actuated micro-beam, where d is the initial width of the gap and z is the vertical displacement of the beam. An external driving force is applied as an electrical driving voltage on the resonator that causes

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[*] Corresponding author. Tel.: +886 4 23924505x7229; fax: +886 4 23924419.
E-mail address: pan1012@ms52.hinet.net (H.-T. Yau).

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doi:10.1016/j.camwa.2010.07.019

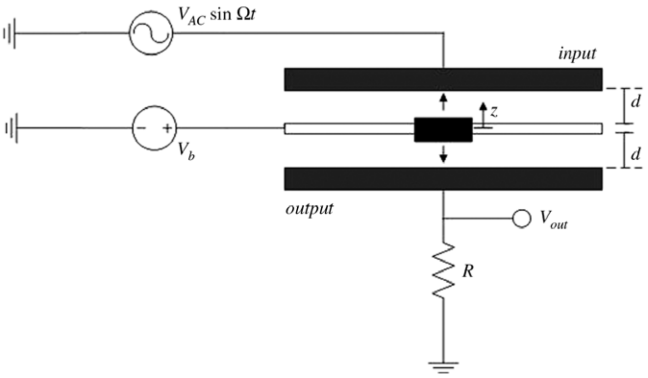

Fig. 1. A schematic diagram of the electrostatically actuated micromechanical resonator.

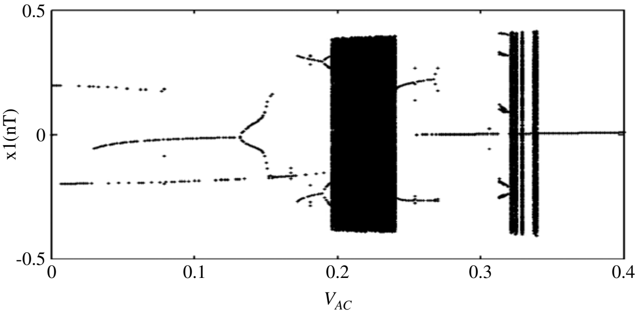

Fig. 2. The bifurcation diagram obtained by varying AC voltage V_{AC} from 0 to 0.4 V.

electrostatic excitation with a dc bias voltage between the electrodes and the resonator: $V_i = V_b + V_{\mathrm{AC}} \cdot \sin\Omega t$, where V_b is the bias voltage and V_{AC} and Ω are the AC amplitude and frequency, respectively. The amplitude of the AC driving voltage is assumed to be much lower than the bias voltage, yielding the nondimensional equation of motion [14]:

$$\ddot{x} + \mu\dot{x} + \alpha x + \beta x^3 = \gamma\left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2}\right) + \frac{A}{(1-x)^2}\sin\omega\tau, \tag{1}$$

where the nondimensional variables x and ω are defined as

$$x = \frac{z}{d}, \qquad \omega = \frac{\Omega}{\omega_0}, \qquad A = 2\gamma\frac{V_{\mathrm{AC}}}{V_b},$$

where ω_0 is the purely elastic natural frequency. Given the states $x_1 = x$, $x_2 = \dot{x}$ and $g(x) = \gamma\left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2}\right)$, this system can be transformed into the following nominal form:

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -\alpha x_1 - \beta x_1^3 - \mu x_2 + g(x_1) + \dfrac{A}{(1-x_1)^2}\sin\omega\tau.\end{cases} \tag{2}$$

This MEMS (2) exhibits complex dynamics and has been studied by Haghighi and Markazi [14] for values of V_{AC} in the range $0 < V_{\mathrm{AC}} < 0.47$ and constant values of $\alpha = 1$, $\beta = 12$, $\gamma = 0.338$, $\mu = 0.01$, $V_b = 3.8$ and $\omega = 0.5$. Fig. 2 displays its bifurcation diagram. In this case, the qualitative behavior of the system is shown against a varying AC voltage from 0 to 0.4. When the AC voltage is increased from zero, periodic motion occurs around one of the center points. Fig. 3 presents the irregular motion that is exhibited by this system at $V_{\mathrm{AC}} = 0.2$ V under initial conditions of $(x_1, x_2) = (0, 0)$. Fig. 3(b) reveals that the corresponding maximum Lyapunov exponent has a positive value, and so the MEMS trajectory is inferred to be in a state of chaotic motion at $V_{\mathrm{AC}} = 0.2$ V. The following section examines the problem of the suppression of chaos of MEMS and introduces the FSMC to cope with this chaotic motion.

3. Robust fuzzy sliding mode control

Consider a chaotic MEMS of the form

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -\alpha x_1 - \beta x_1^3 - \mu x_2 + g(x_1) + \dfrac{A}{(1-x_1)^2}\sin\omega\tau + \Delta f(x_1, x_2) + u,\end{cases} \tag{3}$$