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An Exp-function method for new N-soliton solutions with arbitrary functions of a $(2+1)$-dimensional vcBK system

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ABSTRACT

In this paper, the Exp-function method is generalized to construct N-soliton solutions of the $(2+1)$-dimensional variable-coefficient Broer–Kaup (vcBK) system. As a result, new and more general 1-soliton, 2-soliton and 3-soliton solutions with arbitrary functions are obtained, from which the uniform formulae of N-soliton solutions are derived. Based on one of the obtained 2-soliton solution, the fission and fusion phenomena between the peakons and solitary waves are investigated. It is shown that the Exp-function method may provide us with a straightforward, effective and alternative mathematical tool for generating N-soliton solutions of nonlinear evolution equations in mathematical physics.

1. Introduction

Since the soliton phenomena were first observed by John Scott Russell in 1834 and the KdV equation was solved by the inverse scattering method in 1967 [1], finding exact solutions of nonlinear evolution equations (NLEEs) has become one of the most exciting and active areas of research investigation. In the past several decades, both mathematicians and physicists have made many significant work in this direction and presented some effective methods, such as Hirota's bilinear method [2], Bäcklund transformation [3], Painlevé expansion [4], tanh method [5] sine–cosine method [6], homogeneous balance method [7], Jacobi elliptic function expansion method [8], Fan sub-equation method [9] and variational iteration method [10].

The Exp-function method [11] proposed by Ji-Huan He and Xu-Hong Wu in 2006 provides us with a straightforward and effective method for obtaining generalized solitary wave solutions and periodic solutions of NLEEs. The method has been applied to many kinds of equations like the double sine-Gordon equation [12], Burgers equation [13], Maccari's system [14], Klein–Gordon equation [15], combined KdV–mKdV equation [16], variant Boussinesq equations [17], Broer–Kaup–Kupershmidt equations [18], variable-coefficient equations [19–21], high-dimensional equations [22–24] and discrete equations [25–27]. Recently, Dai et al. [28–30] generalized the Exp-function method to solve stochastic equations. Zhang [31] improved the Exp-function method to obtain not only generalized solitary wave solutions and periodic solutions but also rational solutions. These studies show that the Exp-function method is straightforward, concise, and its applications are promising.

As Wazwaz [32] pointed out that the investigation of N-soliton solutions of NLEEs is a new direction in nonlinear science. With the development of computer science, recently, directly constructing N-soliton solutions have attracted much attention. This is due to the availability of symbolic computation systems like *Mathematica* or *Maple* which enable us to perform the complex and tedious computation on computers. Dai et al. [33] obtained some 3-soliton solutions to the

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$(2+1)$-dimensional potential Boiti–Leon–Manna–Pempinelli (BLMP) equation via the three-wave method. Marinakis [34] generalized the Exp-function method and obtained 1-soliton, 2-soliton and 3-soliton solutions of the famous KdV equation. Zhang and Zhang [35] obtained N-soliton solutions for the KdV equation with variable coefficients by further improving Marinakis' work. In a more recent work [36], with the help of Hermite transformation and white noise theory, Dai and Zhang extended the Exp-function method to the Wick-type stochastic KdV equation and found a 2-soliton solution.

In the present paper, we would like to extend the Exp-function method for constructing new and more general N-soliton solutions of the $(2+1)$-dimensional vcBk system [37]

$$u_{yt} - \alpha(t)[u_{xxy} - 2(uu_x)_y - 2v_{xx}] = 0,$$
$$v_t + \alpha(t)[v_{xx} + 2(uv)_x] = 0, \tag{1}$$

where $\alpha(t)$ is an arbitrary nonzero function of time t. It is evident that when $\alpha(t) = 1$, system (1) becomes the well known (2+1)-dimensional Broer–Kaup (BK) system, which may be derived from the inner-parameter-dependent symmetry constraint of the Kadomtsev–Petviashvili (KP) model [38]. When $\alpha(t) = -1$, system (1) becomes the $(2+1)$-dimensional dispersive water-wave system [39]. When $y = x$, system (1) is reduced to the usual $(1+1)$-dimensional BK system, which is often used to describe the propagation of long waves in shallow water [40]. Using some suitable dependent and independent variable transformations, Chen and Li [41] proved that system (1) can be further transformed to the three dispersive long wave equation and three dimensional Ablowitz–Kaup–Newell–Segur (AKNS) system.

The rest of this paper is organized as follows. In Section 2, we give the description of the Exp-function method [35] for constructing N-soliton solutions of NLEEs. In Section 3, we generalize and use the method [35] to solve system (1) by introducing a new and more general ansatz. In Section 4, some conclusions and discussions are given.

2. Basic idea of the Exp-function method for *N*-soliton solutions of NLEEs

In this section, we recall the Exp-function method [35] for N-soliton solutions of NLEEs. For a given NLEE, say, in two variables x and t:

$$P(u, u_t, u_x, u_{tx}, u_{tt}, u_{xx}, \ldots,) = 0, \tag{2}$$

the Exp-function method for 1-soliton solution is based on the assumption:

$$u(x,t) = \frac{\sum\limits_{i_1=0}^{p_1} a_{i_1} e^{i_1\xi_1}}{\sum\limits_{j_1=0}^{q_1} b_{j_1} e^{j_1\xi_1}}, \quad \xi_1 = k_1 x + c_1 t + w_1, \tag{3}$$

where a_{i_1}, b_{j_1}, k_1, c_1 and w_1 are unknown constants, the values of p_1 and q_1 can be determined by balancing the linear term of highest order in Eq. (2) with the highest order nonlinear term.

In order to seek N-soliton solutions for integer $N > 1$, we generalize Eq. (3) to the following form:

$$u(x,t) = \frac{\sum\limits_{i_1=0}^{p_1}\sum\limits_{i_2=0}^{p_2}\cdots\sum\limits_{i_N=0}^{p_N} a_{i_1 i_2\cdots i_N} e^{\sum\limits_{g=1}^{N} i_g\xi_g}}{\sum\limits_{j_1=0}^{q_1}\sum\limits_{j_2=0}^{q_2}\cdots\sum\limits_{j_N=0}^{q_N} b_{j_1 j_2\cdots j_N} e^{\sum\limits_{g=1}^{N} j_g\xi_g}}, \quad \xi_g = k_g x + c_g t + w_g, \tag{4}$$

given the value of $N = 2$, it becomes:

$$u(x,t) = \frac{\sum\limits_{i_1=0}^{p_1}\sum\limits_{i_2=0}^{p_2} a_{i_1 i_2} e^{\sum\limits_{g=1}^{2} i_g\xi_g}}{\sum\limits_{j_1=0}^{q_1}\sum\limits_{j_2=0}^{q_2} b_{j_1 j_2} e^{\sum\limits_{g=1}^{2} j_g\xi_g}}, \tag{5}$$

which can be used to construct 2-soliton solution.

When $N = 3$, Eq. (4) changes into:

$$u(x,t) = \frac{\sum\limits_{i_1=0}^{p_1}\sum\limits_{i_2=0}^{p_2}\sum\limits_{i_3=0}^{p_3} a_{i_1 i_2 i_3} e^{\sum\limits_{g=1}^{3} i_g\xi_g}}{\sum\limits_{j_1=0}^{q_1}\sum\limits_{j_2=0}^{q_2}\sum\limits_{j_3=0}^{q_3} b_{j_1 j_2 j_3} e^{\sum\limits_{g=1}^{3} j_g\xi_g}}, \tag{6}$$

which can be used to obtain 3-soliton solution.