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The effect of control strength on the synchronization in pinning control questions

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ABSTRACT

This paper investigates the effects of control strength on nonlinearly coupled systems in the process of synchronization, where the coupling strength is an invariable constant. Under the assumption of an asymmetric and reducible coupling matrix, two comparable sufficient conditions are obtained by using the Lyapunov direct method. Moreover, a rough bound for the control strength is presented. A simple simulation is also given to show the validity of the theorems. This work improves the current results that we have.

1. Introduction

Pinning control, as a feasible strategy, could drive networks of coupled oscillators onto some desired trajectory. Generally speaking, we apply pinning control by adding a feedback control input on a fraction of a networks nodes. Though we just exert the direct control on such pinned nodes, it can be propagated to the rest of the oscillators through the coupling among the nodes.

Studying local pinning controllability of a complex, in fact, is equivalent to analyzing the local synchronization of such a network [1], whose methods mainly depend on the master stability function (MSF) [2]. On the basis of [1], a similar method has been applied to the local pinning controllability of a discrete-time case [3].

In the real world, the networks are large-scale or even huge-scale ones i.e. social networks, the Internet network, electric power grid networks, neutral networks, and so on. Therefore, the local synchronization cannot meet our needs, commonly. But, mathematically, the aim of studying local synchronization is to find out about and deal with global and exponential synchronization more easily and deeply. Naturally, we will try to extend the results of the MSF method to global network pinning controllability [4,5]. It proves that global exponential synchronization of error dynamics could be realized through the common Lyapunov stability theory, when the uncoupled function $f(\cdot) \in \mathrm{QUAD}(\Delta, P, \eta)$. Many chaotic systems, such as the Chen chaotic system, the Lü system, the unified chaotic system, and so on, all satisfy this, although the conditions seem rather strict. Hence, this should be endowed with significant mathematical meaning. In [4,5], two adaptive pinning control methods were used to analyze the synchronization of augmented systems.

As for pinning controllability of complex networks, the current works can be roughly sorted into the following groupings. First, pinning synchronization of delayed dynamical networks was discussed in [6,7]. [6] takes advantage of a periodically intermittent control strategy and [7] makes a neural network have imposed control. Next, the optimization and robustness of pinning control were probed in [8]. And then, other applied methods like impulsive control were used in [9]. Also, discrete-time complex networks were also discussed in [9], etc. Finally, global synchronization of fractional-order complex networks was considered in [10] by means of eigenvalue analysis and fractional-order stability theory.

Generally, previous works require the coupling strength c to be large so that the global synchronization of complex networks can be realized. However, there exists a drawback as c becomes larger. This equivalently makes all weights larger

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simultaneously. This must raise the synchronization cost. In this paper, we show that, as a parameter, $\epsilon(t) > 0$ can be used to complete the task with a lower cost.

2. Preliminaries and model description

Suppose that the nonlinearly coupled network is

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = f(x_i(t), t) + c\sum_{j=1}^{m} a_{ij} g(x_j(t)), \quad i = 1, \dots, m, \tag{1}$$

where $x_i(t) = [x_i^1(t), \dots, x_i^m(t)]^T \in R^n$ is the state variable of the ith node, $t \in [0, +\infty)$ is the continuous time, $f : R^n \times [0, +\infty) \to R^n$ is a continuous map, $g(\cdot)$ is some nonlinear function reflecting the nonlinear coupling relationship between those nodes and satisfying that $0 < \beta \le (g(u)-g(v))/(u-v)$ for any $u \ne v \in R$, $A = (a_{ij})_{m\times m}$ is the corresponding coupling matrix that satisfies $a_{ij} \ge 0$ $(i \ne j)$, denoting the coupling coefficients, and $a_{ii} = -\sum_{j=1,j\ne i}^{m} a_{ij}$ for $i, j = 1, \dots, m$, and c is the coupling strength and will be fixed in this paper.

Suppose $s(t)$ is a solution for the uncoupled system, that is

$$\dot{s}(t) = f(s(t), t). \tag{2}$$

We will prove that the coupled network with a single controller shown in (3) can pin the complex dynamical network (1) to $s(t)$:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = f(x_1(t), t) + c\displaystyle\sum_{j=1}^{m} a_{1j} g(x_j(t)) - c\epsilon(t)(g(x_1(t)) - g(s(t))) \\ \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} = f(x_i(t), t) + c\displaystyle\sum_{j=1}^{m} a_{ij} g(x_j(t)), \quad i = 2, \dots, m \end{cases} \tag{3}$$

where $\epsilon(t) > 0$ and $\dot{\epsilon}(t) = \sum_{i=1}^{m_1} \delta x_i^T(t) P \delta x_i(t)$.

Define $\delta x_i(t) = x_i(t) - s(t)$ and $\delta g(x_i(t)) = g(x_i(t)) - g(s(t))$; then the system (3) can be shown as

$$\frac{\delta \mathrm{d}x_i(t)}{\mathrm{d}t} = f(x_i(t), t) - f(s(t), t) + c\sum_{j=1}^{m} \tilde{a}_{ij} \delta g(x_j(t)), \quad i = 1, \dots, m, \tag{4}$$

where $\tilde{a}_{11} = a_{11} - \epsilon(t)$ and $\tilde{a}_{ij} = a_{ij}$ otherwise.

Now, we introduce some definitions, assumptions, and lemmas that will be required throughout the paper.

Assumption 1. Assume that the coupling matrix A is symmetric and reducible, and has the following Perron–Frobenius normal form:

$$A = \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ A_{21} & A_{22} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ A_{p1} & A_{p2} & \cdots & A_{pp} \end{bmatrix} \tag{5}$$

where $A_{ii} \in R^{m_i \times m_i}$ are irreducible, $A_{ij} \ne 0$ $(i > j)$ and $m_1 + m_2 + \cdots + m_p = m$. Without loss of generality, we take $p = 2$.

Definition 1 ([11]). The function $f(\cdot) \in \mathrm{QUAD}(P, \Delta, \eta)$ if there exists a positive definite diagonal matrix $P = \mathrm{diag}(p_1, \dots, p_n)$, a diagonal matrix $\Delta = \mathrm{diag}(\Delta_1, \dots, \Delta_n)$ and a scalar $\eta > 0$ such that

$$(x - y)^T P(f(x) - f(y) - \Delta x + \Delta y) \le -\eta (x - y)^T (x - y)$$

holds for any $x, y \in R^n$.

Lemma 1 ([12]). *Suppose the coupling matrix A has the form (5); then:*

1. $\mathbf{1} = (1, \dots, 1)^T$ *is the right eigenvector of A corresponding to eigenvalue 0 with multiplicity 1, and the real parts of other eigenvalues are negative;*
2. *the left eigenvector ξ of A corresponding to eigenvalue 0 has the following properties: $\xi = (\hat{\xi}^T, 0, \dots, 0)^T$, where $\hat{\xi} = (\xi_1, \dots, \xi_{m_1})^T > 0$ with $\sum_{i=1}^{m_1} \xi_i = \alpha$; for convenience, we write $\Xi = \mathrm{diag}\{\xi_1, \dots, \xi_{m_1}\}$.*

Lemma 2 ([13]). *Suppose that the matrix $A = (a_{ij})_{n\times n}$ satisfies $a_{ij} = a_{ji} \ge 0$ $(i \ne j)$, and $a_{ii} = -\sum_{j=1,j\ne i}^{n} a_{ij}$, $i, j = 1, \dots, n$. Then for any two vectors $x = (x_1, \dots, x_n)^T$ and $y = (y_1, \dots, y_n)^T$, we have*

$$x^T A y = -\sum_{j>i} a_{ij} (x_j - x_i)(y_j - y_i).$$