Quaternary Science Reviews 135 (2016) 103–114

Contents lists available at ScienceDirect

Quaternary Science Reviews

journal homepage: www.elsevier.com/locate/quascirev

Shallow ice approximation, second order shallow ice approximation, and full Stokes models: A discussion of their roles in palaeo-ice sheet modelling and development

N. Kirchner [a, b, *], J. Ahlkrona [a, c], E.J. Gowan [a, b], P. Lötstedt [a, c], J.M. Lea [a, b], R. Noormets [d], L. von Sydow [a, c], J.A. Dowdeswell [e], T. Benham [e]

[a] Bolin Centre for Climate Research, Stockholm University, 106 91 Stockholm, Sweden
[b] Department of Physical Geography, Stockholm University, 106 91 Stockholm, Sweden
[c] Department of Information Technology, Division of Scientific Computing, Uppsala University, Box 337, 751 05 Uppsala, Sweden
[d] Department of Arctic Geology, The University Center in Svalbard (UNIS), P.O. Box 156, 9171 Longyearbyen, Norway
[e] Scott Polar Research Institute, University of Cambridge, Cambridge CB2 1ER, England, United Kingdom

ARTICLE INFO

Article history:
Received 24 March 2015
Received in revised form
10 January 2016
Accepted 19 January 2016
Available online 6 February 2016

Keywords:
Ice sheet models
Palaeo-ice sheets
Simulations
Svalbard
Full Stokes models
Shallow ice approximation
ISCAL method

ABSTRACT

Full Stokes ice sheet models provide the most accurate description of ice sheet flow, and can therefore be used to reduce existing uncertainties in predicting the contribution of ice sheets to future sea level rise on centennial time-scales. The level of accuracy at which millennial time-scale palaeo-ice sheet simulations resolve ice sheet flow lags the standards set by Full Stokes models, especially, when Shallow Ice Approximation (SIA) models are used. Most models used in paleo-ice sheet modeling were developed at a time when computer power was very limited, and rely on several assumptions. At the time there was no means of verifying the assumptions by other than mathematical arguments. However, with the computer power and refined Full Stokes models available today, it is possible to test these assumptions numerically. In this paper, we review (Ahlkrona et al., 2013a) where such tests were performed and inaccuracies in commonly used arguments were found. We also summarize (Ahlkrona et al., 2013b) where the implications of the inaccurate assumptions are analyzed for two paleo-models – the SIA and the SOSIA. We review these works without resorting to mathematical detail, in order to make them accessible to a wider audience with a general interest in palaeo-ice sheet modelling. Specifically, we discuss two implications of relevance for palaeo-ice sheet modelling. First, classical SIA models are less accurate than assumed in their original derivation. Secondly, and contrary to previous recommendations, the SOSIA model is ruled out as a practicable tool for palaeo-ice sheet simulations. We conclude with an outlook concerning the new Ice Sheet Coupled Approximation Level (ISCAL) method presented in Ahlkrona et al. (2016), that has the potential to match the accuracy standards of full Stokes model on palaeo-timescales of tens of thousands of years, and to become an alternative to hybrid models currently used in palaeo-ice sheet modelling. The method is applied to an ice sheet covering Svalbard.

1. Introduction

Simulation of palaeo-ice sheet dynamics provides critical context for modelling the contemporary and potential future behaviour of the Greenland and the Antarctic Ice Sheet. Direct observational data, however, are only available for the later part of the 20th century, when comprehensive satellite remote sensing of the polar regions began. Satellite-observed ice-sheet behaviour suffers from noisiness on inter-annual to sub-annual scale (Howat et al., 2010), and does not capture ice-sheet processes with decadal to millennial timescale lagged responses (Alley et al., 2005; Phillips et al., 2010). This has led to considerable uncertainty in determining the causes of contemporary observed ice-sheet changes. To adequately account for centennial to millennial

---

* Corresponding author. Bolin Centre for Climate Research, and Department of Physical Geography, Stockholm University, 106 91 Stockholm, Sweden.

E-mail addresses: nina.kirchner@natgeo.su.se (N. Kirchner), josefin.ahlkrona@it.uu.se (J. Ahlkrona), evan.gowan@natgeo.su.se (E.J. Gowan), perl@it.uu.se (P. Lötstedt), james.lea@natgeo.su.se (J.M. Lea), riko.noormets@unis.no (R. Noormets), lina@it.uu.se (L. von Sydow), jd16@cam.ac.uk (J.A. Dowdeswell), tbj52@cam.ac.uk (T. Benham).

http://dx.doi.org/10.1016/j.quascirev.2016.01.013
0277-3791/

timescale lags, model validation is required over palaeo-timescales of at least thousands of years, rather than the modern observational period alone (Stokes et al., 2015). Efficient computational methods are needed for repeated simulation of the ice sheet dynamics for calibration of model parameters with observations.

Palaeo ice sheet simulations are also used to support spatial reconstructions of former ice sheets (Stokes et al., 2015), where they bring together modelers and geoscientists. From our experience of discussing results of palaeo ice sheet simulations in a broader group, we found it helpful to be able to communicate aspects of ongoing ice sheet model development without resorting to mathematical detail. It is in such a framework that we here share a description of the roles of some ice sheet models used in palaeo-ice sheet modeling with an audience that has a common interest in the dynamics of former ice sheets.

A full Stokes model is an accurate differential equation model that includes all relevant ice flow dynamics. Such a model is considered to be the most accurate available, capable of describing highly dynamic ice sheets, including ice streams, ice shelves, and grounding line migration, and solutions obtained with Full Stokes models often agree well with data where available, or can be used in inverse models to identify unknown boundary conditions such as e.g. sliding parameters (Isaac et al., 2015). However, their application to palaeo-ice sheet evolution over large domains and for long time intervals is not yet possible because their high level of physical model accuracy requires substantial computational resources (time and memory). For ice sheet scale simulations, computational demands currently restrict the application of full Stokes models to sub-millennial timescale (though normally significantly shorter) simulations of Greenland, and to parts of West Antarctica (Gillet-Chaulet et al., 2012; Gladstone et al., 2012; Joughin et al., 2014; Nowicki et al., 2013; Seddick et al., 2012). To model ice sheet behaviour on longer timescales, approximations to the full Stokes model are used.

Approximations are achieved by discarding small terms in the full Stokes equations, and different levels of approximation are possible depending on how the equations are simplified. One way of identifying the small terms is to scale the variables with a small parameter as in the Shallow Ice Approximation (SIA) and Second Order Shallow Ice Approximation (SOSIA) models.

Classical SIA models are prime examples of approximate ice sheet models (see Section 2) that can be run on glacial/interglacial timescales. They are widespread because they allow, in principle, for model validation over centennial to millennial timescales. Since the 1980's, SIA models have been used for palaeo-glaciological simulations of the ice sheets of the Quaternary (Hughes, 1981; Holmlund and Fastook, 1995; Siegert, 1997; Ritz et al., 1997; Tarasov and Peltier, 2000; Kirchner et al., 2011a; Kusahara et al., 2015). Despite their simplicity, results from SIA models should be evaluated carefully because terms that were assumed to be small in the model derivation (and are hence neglected) may in fact be large in some situations and have a major influence on the solution. It is difficult to tell in advance when and where this will happen, i.e. where a SIA model will become inaccurate. The discrepancy between models can be estimated in well-designed test problems (Pattyn et al., 2012; 2013) and help to guide a user to decide in which situations the simple model is applicable. Furthermore, because SIA models are approximate models, they capture only bulk ice sheet flow, and therefore lack the ability to adequately simulate other ice dynamic behaviour, such as fast-flowing ice streams, the transition of ice-sheets into ice-shelves with associated grounding-line migration, ice margins and ice domes (Hutter, 1983; Gudmundsson, 2003; Blatter et al., 2011; Kirchner et al., 2011b; Schoof and Hewitt, 2013). This is highly problematic, since recent spatial reconstructions indicate that palaeo-ice sheets were more dynamic than previously thought, with changing ice-stream behaviours, shifting domes and ice-margin dynamics all playing crucial roles in their evolution (Ingólfsson and Landvik, 2012; Ó Cofaigh et al., 2013; Stokes et al., 2015). In combination with records from geological archives holding evidence of former rapid ice stream retreat and subsequent ice sheet decay, models that are capable of simulating highly dynamic palaeo-ice sheets will advance our understanding of ice sheet disintegration, and the spatial and temporal scales involved, both in the past and in the future.

The limitations of the SIA have long since been known, and other approximate models for grounded ice flow have been constructed to overcome these issues. Such models typically belong to one of the following categories: 1) higher order extensions of the SIA model, such as the SOSIA model, 2) so-called hybrid models, which heuristically combine different lower order models (Bueler and Brown, 2009; www.pism-docs.org; Pollard and DeConto, 2012), and 3) more complex higher-order models such as the Blatter-Pattyn model (Blatter, 1995; Pattyn, 2003). These approximate models vary in complexity and accuracy and are briefly described in Section 2. The weaknesses of the simplified models can be examined by comparing the solutions with full Stokes solutions in carefully chosen test problems. From a theoretical-conceptual viewpoint, SOSIA was thought to represent a powerful tool to perform palaeo-ice sheet simulations (Baral et al., 2001), and numerical implementations of which could become an alternative to the hybrid models (Kirchner et al., 2011b). However, in Ahlkrona et al. (2013a,b), a mathematical and computational analysis of SOSIA was carried out which showed that this is not the case. This is an example for numerical model assumption validation that only became possible with advanced computing power and refined Stokes models.

Our objective is to discuss different levels of approximation of the full Stokes equations, in particular in the SIA and SOSIA models, without mathematical and computational details and based on the results in Ahlkrona et al. (2013a,b). A small scale ε is introduced and terms proportional to certain powers of ε are ignored in SIA. Terms multiplied by higher powers of ε are incorporated in SOSIA thus making SOSIA formally more accurate but not in reality as shown in Ahlkrona et al. (2013b). A boundary layer at the surface of the ice is not captured correctly by SOSIA. The scaling has to be introduced in a different way in the boundary layer. As SIA models continue to be widespread, accuracy and applicability of SIA are important subjects. A method to quantify the deviation of SIA from the full Stokes model is proposed in Ahlkrona et al. (2016). There the SIA is chosen locally if the modeling error is sufficiently small while otherwise the full Stokes equations are solved. This method is referred to as ISCAL (Ice Sheet with Coupled Approximation Levels), and is designed as a complementary approach to the hybrid ice sheet models already available for palaeo-ice sheet simulations. The method is applied to an ice sheet covering Svalbard in an example.

2. Higher order ice sheet models

During the last decade, several ‘new generation’ ice sheet modelling techniques have been presented that aim to provide more accurate results than obtained from SIA, by incorporating neglected aspects of ice sheet dynamics and mostly predicting ice sheet behaviour on future, centennial timescales (Cornford et al., 2013; Gillet-Chaulet et al., 2012; Larour et al., 2012; Peyaud et al., 2007; Pollard and DeConto, 2012; Sato and Greve, 2012; Winkelman et al., 2011). It is commonly suggested that a model hierarchy can be established based on increasing physical accuracy. This can, for instance, be expressed by the ‘approximate order’ of a model (n): from the zero-order models such as the classical SIA and