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Positive periodic solutions for the neutral ratio-dependent predator–prey model[☆]

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ABSTRACT

By using a continuation theorem based on coincidence degree theory, we obtain some new sufficient conditions for the existence of positive periodic solutions for the neutral ratio-dependent predator–prey model with Holling type II functional response.

1. Introduction

The dynamic relationship between predator and its prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. The traditional predator–prey models have been studied extensively (See, for example, [1–4] and references cited therein).

Recently, there is a growing explicit biological and physiological evidences that a more suitable general predator–prey theory should be based on the so-called ratio-dependent theory. This is strongly supported by numerous field and laboratory experiments and observations. Based on the Michaelis–Menten or Holling type II function, Arditi and Ginzburg [5] proposed the ratio-dependent predator–prey model. Subsequently, many authors [5,6] have observed that the system exhibits much richer, more complicated, and more reasonable or acceptable dynamics. Beretta and Kuang [6] introduced a single discrete time delay into the predator equation in the foregoing model. In view of the periodicity of the actual environment, Fan and Wang [7] established verifiable criteria for the global existence of positive periodic solutions of a more general delayed ratio-dependent predator–prey model with periodic coefficients. Kuang [8] studied the local stability and oscillation of the following neutral delay Gause-type predator–prey system.

In this paper, motivated by the above work, we shall consider the following neutral delay ratio-dependent predator–prey model with Holling type II functional response

$$\begin{cases} x'(t) = x(t)[a(t) - bx(t-\sigma_1) - \rho x'(t-\sigma_2)] - \dfrac{c(t)x(t)y(t)}{my(t)+x(t)}, \\ y'(t) = y(t)\left[-d(t) + \dfrac{f(t)x(t-\tau)}{my(t-\tau)+x(t-\tau)}\right]. \end{cases} \tag{1.1}$$

As pointed out by Kuang [9], it would be of interest to study the existence of periodic solutions for periodic systems with time delay. To our knowledge, no such work has been done on the global existence of positive periodic solutions of (1.1). Our aim in this paper is, by using the coincidence degree theory developed by Gaines and Mawhin [10], to derive a set of easily verifiable sufficient conditions for the existence of positive periodic solutions of system (1.1).

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For convenience, we will use the notations: $|r|_0 = \max_{t\in[0,\omega]}\{|r(t)|\}$, $\bar{r} = \frac{1}{\omega}\int_0^\omega r(t)\mathrm{d}t$, $\hat{r} = \frac{1}{\omega}\int_0^\omega |r(t)|\mathrm{d}t$, where $r(t)$ is a continuous ω-periodic function.

In this paper, we make the following assumptions for system (1.1).

(H_1) $m, b, \rho \in (0, \infty)$; $\sigma_1, \sigma_2, \tau \in R$; $a \in C(R, R)$, $c, d, f \in C(R, [0, +\infty))$ are ω-periodic functions; $\bar{a} > 0, \bar{d} > 0, \bar{f} > 0$.
(H_2) $\rho e^B < 1$, where $B = \ln A + A + (\hat{a} + \bar{a})\omega$ and $A = \frac{(1+\rho)\bar{a}}{b}$.
(H_3) $\bar{c} < m\bar{a}$.
(H_4) $\bar{d} < \bar{f}$.

2. Existence of positive periodic solution

In this section, we shall study the existence of at least one positive periodic solution of system (1.1). The method to be used in this paper involves the applications of the continuation theorem of coincidence degree. For the readers' convenience, we first introduce a few concepts and results about the coincidence degree.

Let X, Z be real Banach spaces, $L : \mathrm{Dom}\, L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping. The mapping L is said to be a Fredholm mapping of index zero, if $\dim\mathrm{Ker}\, L = \mathrm{codim}\mathrm{Im}\, L < +\infty$ and $\mathrm{Im}\, L$ is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$, such that $\mathrm{Im}\, P = \mathrm{Ker}\, L$, $\mathrm{Ker}\, Q = \mathrm{Im}\, L = \mathrm{Im}\,(I - Q)$. It follows that the restriction L_P of L to $\mathrm{Dom}\, L \cap \mathrm{Ker}\, P : (I - P)X \to \mathrm{Im}\, L$ is invertible. Denote the inverse of L_P by K_P. The mapping N is said to be L-compact on $\overline{\Omega}$, if Ω is an open bounded subset of X, $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}\, Q$ is isomorphic to $\mathrm{Ker}\, L$, there exists an isomorphism $J : \mathrm{Im}\, Q \to \mathrm{Ker}\, L$.

Lemma 2.1 (*Continuation Theorem [10, p. 40]*). *Let $\Omega \subset X$ be an open bounded set, L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Suppose* (i) *for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \mathrm{Dom}\, L$, $Lx \neq \lambda Nx$;* (ii) *for each $x \in \partial\Omega \cap \mathrm{Ker}\, L$, $QNx \neq 0$;* (iii) $\deg\{JQN, \Omega \cap \mathrm{Ker}\, L, 0\} \neq 0$. *Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}\, L$.*

Lemma 2.2. *Assume that $h(t), g(t)$ are continuous and nonnegative functions defined on the interval $[\alpha, \beta]$. Then there exists $\xi \in [\alpha, \beta]$ such that $\int_\alpha^\beta h(t)g(t)\mathrm{d}t = h(\xi)\int_\alpha^\beta g(t)\mathrm{d}t$.*

We are now in a position to state and prove our main result.

Theorem 2.1. *Assume that* (H_1)–(H_4) *hold. Then system (1.1) has at least one ω-periodic solution with strictly positive components.*

Proof. Consider the following system:

$$\begin{cases} u_1{}'(t) = a(t) - be^{u_1(t-\sigma_1)} - \rho e^{u_1(t-\sigma_2)}u_1{}'(t-\sigma_2) - \dfrac{c(t)e^{u_2(t)}}{me^{u_2(t)} + e^{u_1(t)}}, \\ u_2{}'(t) = -d(t) + \dfrac{f(t)e^{u_1(t-\tau)}}{me^{u_2(t-\tau)} + e^{u_1(t-\tau)}}, \end{cases} \tag{2.1}$$

where all functions are defined as ones in system (1.1). It is easy to see that if system (2.1) has one ω-periodic solution $(u_1^*(t), u_2^*(t))^T$, then $(x^*(t), y^*(t))^T = (e^{u_1^*(t)}, e^{u_2^*(t)})^T$ is a positive ω-periodic solution of system (1.1). Therefore, to complete the proof, it suffices to show that system (2.1) has at least one ω-periodic solution.

Take $X = \{u = (u_1(t), u_2(t))^T \in C^1(R, R^2) : u_i(t+\omega) = u_i(t), t \in R, i = 1, 2\}$, $Z = \{u = (u_1(t), u_2(t))^T \in C(R, R^2) : u_i(t+\omega) = u_i(t), t \in R, i = 1, 2\}$ and denote $|u|_\infty = \max_{t\in[0,\omega]}\{|u_1(t)| + |u_2(t)|\}$, $\|u\| = |u|_\infty + |u'|_\infty$. Then X and Z are Banach spaces when they are endowed with the norms $\|\cdot\|$ and $|\cdot|_\infty$, respectively. Let $L : X \to Z$ and $N : X \to Z$ be $L(u_1(t), u_2(t))^T = (u_1{}'(t), u_2{}'(t))^T$ and

$$N\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} a(t) - be^{u_1(t-\sigma_1)} - \rho e^{u_1(t-\sigma_2)}u_1{}'(t-\sigma_2) - \dfrac{c(t)e^{u_2(t)}}{me^{u_2(t)} + e^{u_1(t)}} \\ -d(t) + \dfrac{f(t)e^{u_1(t-\tau)}}{me^{u_2(t-\tau)} + e^{u_1(t-\tau)}} \end{bmatrix}.$$

With these notations, system (2.1) can be written in the form $Lu = Nu$, $u \in X$. Obviously, $\mathrm{Ker}\, L = R^2$, $\mathrm{Im}\, L = \left\{(u_1(t), u_2(t))^T \in Z : \int_0^\omega u_i(t)\mathrm{d}t = 0, i = 1, 2\right\}$ is closed in Z, and $\dim\mathrm{Ker}\, L = \mathrm{codim}\mathrm{Im}\, L = 2$. Therefore L is a Fredholm mapping of index zero. Now define two projectors $P : X \to X$, $Q : Z \to Z$ as

$$P\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix}, \quad \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \in X, \qquad Q\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix}, \quad \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \in Z.$$