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Stochastic stability of impulsive BAM neural networks with time delays

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ABSTRACT

The problem of stability analysis of stochastic BAM neural networks with time delays and impulse effects is investigated in this paper. Using the Lyapunov technique and the generalized Hanalay inequality, we characterize theoretically the aggregated effects of impulse and stability properties of the impulse-free version. The present approaches allow us to estimate the feasible upper bounds of impulse strengths and can also extend to the more general impulsive nonlinear systems with delays.

1. Introduction

Bidirectional associative memory (BAM) neural networks were proposed by Kosko [1,2]. Because of good application prospects in pattern recognition, signal and image processing, BAM model has attracted much attention. It is well known that stability analysis of neural networks is a prerequisite for the practice design and applications [3,4]. Recently, its analog with impulse and delay has been becoming a research focus due to the fact that the evolutionary processes of neural networks including BAM are usually characterized by abrupt changes at certain time in axonal transmission [5–7]. As is well known, in real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain stochastic inputs [7,8]. Hence, the stability analysis problem for stochastic neural networks becomes increasingly significant, and some results related to this problem have recently been published [9–12]. However, to the best of the authors' knowledge, few results for the stochastic stability analysis on BAM with delay and impulse have been reported in the literature.

2. Problem statement and preliminaries

In this paper, we consider the following impulsive BAM neural networks with time delays and stochastic perturbation:

$$\begin{aligned}
&\mathrm{d}x(t) = [-C_1x(t) + A_1f(x(t)) + B_1f(y(t-\tau))]\mathrm{d}t + [W_{01}x(t) + W_{11}y(t-\tau)]\mathrm{d}W(t),\\
&\mathrm{d}y(t) = [-C_2y(t) + A_2f(y(t)) + B_2f(x(t-\tau))]\mathrm{d}t + [W_{02}y(t) + W_{12}x(t-\tau)]\mathrm{d}W(t), \quad t \neq t_k,\\
&\Delta x(t_k) = R_{1k}x(t_k^-), \qquad \Delta y(t_k) = R_{2k}y(t_k^-), \quad k = 1, 2, \ldots
\end{aligned} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$ and $y(t) = [y_1(t), \ldots, y_n(t)]^T \in R^n$ denote the state vector of neurons. The first two parts of (1), named continuous component of (1), are the continuous parts of model (1), which describe the continuous evolution processes of BAM. $C_i = \mathrm{diag}\,(c_{i1}, \ldots, c_{in}) > 0$, $A_i = (a_{ij}^{(i)})_{n\times n}$, $B_i = (b_{ij}^{(i)})_{n\times n}$, $W_{0i} = (w_{ij}^{(0i)})_{n\times n}$, $W_{1i} = (w_{ij}^{(1i)})_{n\times n}$ and $f(u) = [f_1(u_1), \ldots, f_n(u_n)]^T \in R^n$. $W(t) = [w_1(t), \ldots, w_n(t)]^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is

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right continuous and F_0 contains all P-null sets). The last two parts are the discrete parts of model (1), which describe the abrupt change of state at the moments of time t_k (called impulsive moments), where $\{t_k\}$ satisfy $0 \le t_0 < t_1 < t_2 < \cdots$ and $\lim_{k\to\infty} t_k = \infty$; $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ and $\Delta\psi(t_k) = y(t_k^+) - y(t_k^-)$ are the impulses at moment t_k. We assume, as usual, that the solution is right continuous at t_k, i.e., $x(t_k) = x(t_k^+)$, $y(t_k) = y(t_k^+)$. Throughout this paper, we make the following assumption:

(H1) The activation function $f_i(\cdot)$ is bounded, and satisfies the Lipschitzian condition, namely, there exist positive scalars l_i $(i = 1, 2, \ldots, n)$ such that $|f_i(\alpha)-f_i(\beta)| \le l_i|\alpha-\beta|$ with $f(0) = 0$, for any $\alpha, \beta \in R$. We denote $L = \mathrm{diag}\,(l_i^f,\ i = 1, \ldots, n)$.

The initial conditions associated with system (1) are of the following equations:

$$x_i(t) = \phi_i(t), \qquad y_j(t) = \psi_j(t), \quad t_0 - \tau \le t \le t_0,$$

in which $\phi_i(t), \psi_j(t)$ $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m)$ are continuous functions.

To conclude this section, we present a generalized Hanalay inequality.

Lemma 1 (*Two-Order Hanalay-Type Inequality for Stochastic Systems*). *Suppose that $u : [t_0-\tau, \infty) \to R^n$, $v : [t_0-\sigma, \infty) \to R^n$ satisfy*

$$\begin{aligned} du(t) &\le [-\alpha_1 u(t) + \beta_1 v(t-\sigma)]dt + \sigma_1(t, u(t), v(t))dW(t),\\ dv(t) &\le [-\alpha_2 v(t) + \beta_2 u(t-\tau)]dt + \sigma_2(t, u(t), v(t))dW(t) \end{aligned}$$

where W is as defined in model (1), $\alpha_1, \alpha_2, \beta_1, \beta_2$ are positive constants satisfying $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$. Then, $E\{u(t)\} \le K_0 e^{-\eta(t-t_0)}$, $E\{v(t)\} \le K_0 e^{-\eta(t-t_0)}$, $t < t_0$, where $K_0 = \max\{E\{\bar u(t_0)\}, E\{\bar v(t_0)\}\}$ with $E\{\bar u(t_0)\} = \sup_{t_0-\tau\le\theta\le t_0} E\{u(\theta)\}$, $E\{\bar v(t_0)\} = \sup_{t_0-\sigma\le\theta\le t_0} E\{v(\theta)\}$, $\eta > 0$ is the largest positive solution satisfying both of the inequalities $\alpha_1 - \eta - \beta_1 e^{\eta\tau} \ge 0$, $\alpha_2 - \eta - \beta_2 e^{\eta\sigma} \ge 0$.

3. Stochastic stability

Theorem 1. *Under assumptions* (H1), *the origin of system* (1) *is globally exponentially stable if the following conditions are satisfied:*

(i) *There exist positive scalars α_i, β_i and ε_i $(i = 1, 2)$ such that*

$$\Omega_{x1} + \alpha_1 I \le 0, \qquad \Omega_{x2} - \beta_1 I \le 0, \qquad \Omega_{y1} + \alpha_2 I \le 0, \qquad \Omega_{y2} - \beta_2 I \le 0$$

where $\Omega_{x1} = -2C_1 + W_{01}^T W_{01} + 2\bar l I + \varepsilon_2 I + \varepsilon_1 \lambda_{\max}(W_{01}^T W_{01}) I$, $\Omega_{x2} = \varepsilon_2^{-1}\bar l^2 I + \varepsilon_1^{-1}\lambda_{\max}(W_{11}^T W_{11}) I + W_{11}^T W_{11}$, $\Omega_{y1} = -2C_2 + W_{02}^T W_{02} + 2\bar l I + \varepsilon_2 I + \varepsilon_1 \lambda_{\max}(W_{02}^T W_{02}) I$, $\Omega_{y2} = \varepsilon_2^{-1}\bar l^2 I + \varepsilon_1^{-1}\lambda_{\max}(W_{12}^T W_{12}) I + W_{12}^T W_{12}$.

(ii) $\tau \le t_j - t_{j-1}, j = 1, 2, \ldots;$ $\eta - \frac{\ln c}{\delta} > 0$ *with* $\delta \ge \frac{k}{t-t_0}$, *for any* $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots$

where $c = \max\{\bar a_k, \bar b_k\}$ with $\bar a_k = \max\{\exp(\eta\tau), a_k\}$ and $\bar b_k = \max\{\exp(\eta\tau), b_k\}$, $\eta > 0$ satisfies $\alpha_1 - \eta - \beta_1 e^{\eta\tau} \ge 0$ and $\alpha_2 - \eta - \beta_2 e^{\eta\tau} \ge 0$.

Proof. Let us consider the following Lyapunov function candidate

$$V(x(t),\ y(t)) = V_1(x(t)) + V_2(y(t)) \tag{2}$$

where $V_1(x(t)) = x^T(t)x(t)$, $V_2(y(t)) = y^T(t)y(t)$.

When $t \in [t_{k-1}, t_k)$, by applying Ito's formula, the stochastic derivative of $V_1(x(t))$ along the solution of (1) can be obtained as follows:

$$\begin{aligned} dV_1(x(t)) &= \{2x^T(t)[-C_1x(t) + A_1 f(x(t)) + B_1 f(x(t-\tau))]\\ &\quad + [W_{01}x(t) + W_{11}y(t-\tau)]^T[W_{01}x(t) + W_{11}y(t-\tau)]\}dt + 2x^T(t)[W_{01}x(t) + W_{11}y(t-\tau)]dW(t)\\ &\le \{x^T(t)[-2C_1 + W_{01}^T W_{01} + 2\bar l I + \varepsilon_2 I + \varepsilon_3\lambda_{\max}(W_{01}^T W_{01})I]x(t)\\ &\quad + y^T(t-\tau)[\varepsilon_2^{-1}\bar l^2 I + \varepsilon_3^{-1}\lambda_{\max}(W_{11}^T W_{11})I + W_{11}^T W_{11}]y(t-\tau)\}dt\\ &\quad + 2x^T(t)[W_{01}x(t) + W_{11}y(t-\tau)]dW(t). \end{aligned}$$

Therefore, for $t \in [t_{k-1}, t_k)$,

$$dV_1(x(t)) \le [-\alpha_1 V_1(x(t)) + \beta_1 V_2(y(t-\tau))]dt + 2x^T(t)[W_{01}x(t) + W_{11}y(t-\tau)]dW(t). \tag{3a}$$

Similarly, for $t \in [t_{k-1}, t_k)$,

$$dV_2(y(t)) \le [-\alpha_2 V_2(y(t)) + \beta_2 V_1(x(t-\tau))]dt + 2y^T(t)[W_{02}y(t) + W_{12}x(t-\tau)]dW(t). \tag{3b}$$

Taking mathematical expectations and then integrating on both sides of (3) and (4), respectively, yield

$$E\{V_1(x(t))\} \le E\{\bar V(t_{k-1})\}\exp(-\eta(t-\tau)), \qquad E\{V_2(y(t))\} \le E\{\bar V(t_{k-1})\}\exp(-\eta(t-\tau)) \tag{4}$$