Available online at www.sciencedirect.com

ScienceDirect

Computers and Mathematics with Applications 54 (2007) 599–616

An International Journal
computers & mathematics with applications

www.elsevier.com/locate/camwa

Classes of multivalent analytic functions involving the Dziok–Srivastava operator

J. Patel[a], A.K. Mishra[b], H.M. Srivastava[c,*]

[a] *Department of Mathematics, Utkal University, Vani Vihar, Bhubaneswar 751004, Orissa, India*
[b] *Department of Mathematics, Berhampur University, Bhanja Bihar, Berhampur 760 007, Orissa, India*
[c] *Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3P4, Canada*

Received 2 August 2006; accepted 30 August 2006

Abstract

The object of the present paper is to investigate some inclusion relationships and a number of other useful properties of several subclasses of multivalent analytic functions, which are defined here by using the Dziok–Srivastava operator. Relevant connections of the results presented here with those obtained in earlier works are pointed out.

Keywords: Analytic functions; Multivalent functions; Gauss hypergeometric function; Generalized hypergeometric functions; Hadamard product (or convolution); Dziok–Srivastava operator; Carlson–Shaffer operator; Ruscheweyh derivative; Briot–Bouquet differential subordination; Cho–Kwon–Srivastava operator; Generalized neighborhood; Choi–Saigo–Srivastava operator

1. Introduction

Let $\mathcal{A}_p$ denote the class of functions normalized by

$$f(z) = z^p + \sum_{k=1}^{\infty} c_{p+k} z^{p+k} \quad (p \in \mathbb{N} := \{1, 2, 3, \ldots\}), \tag{1.1}$$

which are *analytic* in the *open* unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

For functions f given by (1.1) and g given by

$$g(z) = z^p + \sum_{k=1}^{\infty} d_{p+k} z^{p+k},$$

* Corresponding author. Tel.: +1 250 472 5692; fax: +1 250 721 8962.
E-mail addresses: jpatelmath@yahoo.co.in (J. Patel), akshayam2001@yahoo.co.in (A.K. Mishra), harimsri@math.uvic.ca (H.M. Srivastava).

0898-1221/$ - see front matter
doi:10.1016/j.camwa.2006.08.041

the Hadamard product (or convolution) of f and g is defined by

$$(f \star g)(z) := z^p + \sum_{k=1}^{\infty} c_{p+k} d_{p+k} z^{p+k} =: (g \star f)(z).$$

Let the functions f and g be analytic in $\mathbb{U}$. We say that the function f is *subordinate* to g, written as $f \prec g$ in $\mathbb{U}$ or

$$f(z) \prec g(z) \quad (z \in \mathbb{U}),$$

if there exists a function w, analytic in $\mathbb{U}$ with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1,$$

such that

$$f(z) = g(w(z)) \quad (z \in \mathbb{U}).$$

It follows that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Longrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

In particular, if g is *univalent* in $\mathbb{U}$, we have the following equivalence (cf., e.g., [1,2]):

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Longleftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, f is said to be subordinate to g in the disk

$$\mathbb{U}_r := \{z : z \in \mathbb{C} \text{ and } |z| < r\},$$

if the function $f_r(z) = f(rz)$ is subordinate to $g_r(z) = g(rz)$ in $\mathbb{U}$. Hence, if $f \prec g$ in $\mathbb{U}$, then $f \prec g$ in $\mathbb{U}_r$ for every r $(0 < r < 1)$.

For complex parameters

$$a_1, \ldots, a_l \quad \text{and} \quad b_1, \ldots, b_m \quad (b_j \notin \mathbb{Z}_0^- := \{0, -1, -2, \ldots\};\ j = 1, \ldots, m),$$

the *generalized hypergeometric function* ${}_lF_m$ is defined (cf., e.g., [3, p. 19 *et seq.*]) by the following infinite series:

$$
\begin{aligned}
&{}_lF_m(a_1, \ldots, a_l; b_1, \ldots, b_m; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_l)_k}{(b_1)_k \cdots (b_m)_k} \frac{z^k}{k!} \\
&(l, m \in \mathbb{N}_0 := \mathbb{N} \cup \{0\};\ l < m + 1 \text{ and } z \in \mathbb{C};\ \ l = m + 1 \text{ and } z \in \mathbb{U};\ \ l = m + 1,\ z \in \partial\mathbb{U}, \text{ and } \Re(\omega) > 0),
\end{aligned}
$$

where

$$\omega := \sum_{j=1}^{m} b_j - \sum_{j=1}^{l} a_j$$

and $(\lambda)_k$ is the Pochhammer symbol (or the *shifted factorial*) defined, in terms of the Gamma function Γ, by

$$(\lambda)_k = \frac{\Gamma(\lambda + k)}{\Gamma(\lambda)} = \begin{cases} 1 & (k = 0), \\ \lambda(\lambda + 1) \cdots (\lambda + k - 1) & (k \in \mathbb{N}). \end{cases}$$

Dziok and Srivastava [4] considered a linear operator

$$H_p(a_1, \ldots, a_l; b_1, \ldots, b_m) : \mathcal{A}_p \longrightarrow \mathcal{A}_p$$

defined by the following Hadamard product:

$$
\begin{aligned}
&H_p(a_1, \ldots, a_l; b_1, \ldots, b_m) f(z) := [z^p\ {}_lF_m(a_1, \ldots, a_l; b_1, \ldots, b_m; z)] \star f(z), \\
&\quad (l \leqq m + 1;\ l, m \in \mathbb{N}_0;\ z \in \mathbb{U}).
\end{aligned} \tag{1.2}
$$