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# Lower bounds for the mixed capacitated arc routing problem ☆

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ABSTRACT

Capacitated arc routing problems (CARP) arise in distribution or collecting problems where activities are performed by vehicles, with limited capacity, and are continuously distributed along some pre-defined links of a network. The CARP is defined either as an undirected problem or as a directed problem depending on whether the required links are undirected or directed. The mixed capacitated arc routing problem (MCARP) models a more realistic scenario since it considers directed as well as undirected required links in the associated network. We present a compact flow based model for the MCARP. Due to its large number of variables and constraints, we have created an aggregated version of the original model. Although this model is no longer valid, we show that it provides the same linear programming bound than the original model. Different sets of valid inequalities are also derived. The quality of the models is tested on benchmark instances with quite promising results.



## 1. Introduction

Capacitated arc routing problems (CARP) arise in distribution or collecting problems where activities are performed by vehicles, with limited capacity, and are continuously distributed along some pre-defined links (roads, streets) of an associated network. The CARP can be either undirected or directed. In the undirected case, the required links can be served in any direction. In the directed case, the required links must be served only in the defined direction. The mixed capacitated arc routing problem (MCARP) models a more realistic scenario as it accommodates simultaneously both types of links. The MCARP is a NP-hard problem since it generalizes the CARP [19] which is known to be NP-hard.

The research on CARP lower bounding procedures, solution and modelling approaches performed in the last decade is surveyed by Wøhlk [30]. Many real world applications, such as household refuse collection, winter gritting, postal distribution, metre reading, street swiping, can be modelled either as a CARP or a MCARP. The surveys on arc routing by Assad and Golden [3], Eiselt et al. [13,14] and Dror's book [11] include many references on real world problems modelled as ARPs until the year 2000. More recent publications on arc routing real world applications include postal delivering by Irnich [20]; a real situation arising on an industrial company by Moreira et al. [25] and garbage collection, which is a main concern of municipalities (see [2,5,10,17,24,26,27]).

The MCARP study reported in this paper is motivated by a household refuse collection problem defined in a quarter of Lisbon. Old town quarters are usually represented by directed graphs, while new town quarters are defined in mixed networks.

Many CARP applications differ on the features of the system collection design, namely the number of depots and its location ([1,10,18,26], to name a few).

An approach to solve capacitated arc routing problems is by means of well known transformations into equivalent node routing problems [29,4,22,5]. The main idea is to use available and well tested methods for node routing problems. However, these transformations lead, in general, to networks that are substantially larger than the originals and many authors prefer to develop models on the original graph. This is also the approach followed on this paper.

The first formulation for the CARP was proposed by Golden and Wong [19] and includes an exponential number of constraints. It is also stated that the exponential sized set of subtour-breaking constraints may be replaced by a more compact set, based on flow variables. The lower bound provided from the LP relaxation of this formulation is known to be equal to zero (see [12]). Golden and Wong [19] did not use the compact model to get lower bounds for the CARP. Instead, a different lower bound method was developed and its bound was shown to be equal to the bound obtained from

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the optimal value of a relaxation where capacity and connectivity constraints are omitted.

A different model for the undirected CARP was proposed, in 1998, by Belenguer and Benavent [6]. In 2003, the same authors [7] suggested a different formulation for the same problem that has only a single variable for each edge of the underlying graph, but it contains an exponential number of constraints. This formulation is shown to be non-valid, similarly to what happens with one of the models presented in this paper.

Later on, Belenguer et al. [8] developed a study on lower bounds for the MCARP based on the model defined in [7]. This non-valid model for the MCARP is similar to models presented for other mixed arc routing problems, as the mixed Chinese postman problem [28] and the mixed general routing problem [9]. The authors use this model and several valid inequalities in a cutting plane fashion to get lower bounds for the MCARP that outperformed the previous best known bounds.

In this paper, we formulate the MCARP by a compact model, and as far as we know, it is the first valid model for the MCARP given in the literature that is tested on reasonable large sized instances. We use two well known ideas to design this formulation for the problem: (i) the concept of flows to guarantee the connectivity of the solutions (see, for instance [15,16]) and (ii) the concept of indexing the variables by vehicle to guarantee a matching between trips and vehicles (see, for instance [23]). The model will be used within an ILP package to solve medium sized problems and to produce lower bounds on larger instances. Lower bounds are also obtained from the corresponding linear programming relaxation.

Our model differs from the model by Golden and Wong [19] in several aspects: (i) it formulates the mixed case while their model was developed for the undirected CARP; (ii) the flow variables have a different interpretation (here they are related with the demands to be served and in their paper flows are associated with the number of edges to serve); (iii) additional constraints are included to ensure that trips start at the depot; and (iv) extra valid inequalities are considered to strengthen the linear programming relaxation. A straightforward extended formulation of Golden and Wong [19] to the mixed CARP was tested on small instances by Lacomme et al. [21]; it also differs from ours on the above mentioned items (ii)–(iv).

Due to the vehicle indexing, the number of variables and constraints in our model is huge. Following the literature on the classic vehicle routing problem (VRP), we may try to get a more compact model, where links and flow variables are not disaggregated by vehicle. Unfortunately, in contrast with the classic VRP, it does not seem easy to find a similar valid model for the MCARP. However, we will present and discuss one such aggregated model which, although not valid, is attractive for three reasons. First, an integer optimal solution of the aggregated model is easier to compute than an optimal integer solution of the original model. An integer solution of the new model gives a lower bound on the optimal solution value of the MCARP which, as our computational experiments will show, provides competitive lower bound values for some classes of well known MCARP instances. Second, for certain instances, the integer lower bound value is equal to the value of a known heuristic solution, thus certifying the optimality of this solution. Finally, we will also prove that the linear programming relaxation values of the two models are equal. This means that the disaggregated model can be replaced by the non-valid model in order to produce the linear programming bound in a much faster way (since the aggregated model has fewer constraints and variables).

Comparing with the Belenguer and Benavent [7] formulation, the main difference between our aggregated model and their model lies on the network type (mixed versus undirected) and on the size of the models since our model is compact and theirs has an exponential number of constraints. That is, in our model capacity and connectivity constraints are enforced by using the additional flow variables and the constraints linking the two sets of variables. In [7] the authors do not use the extra set of variables but use, in turn, exponential sized sets of constraints to force connectivity.

The paper is organized as follows. In Section 2, we define the MCARP, set notation and present the two formulations, the valid and the non-valid formulation. We also prove that both formulations produce the same linear programming bound and discuss valid inequalities. Section 3 reports the results from the computational experiments on a set of known benchmark instances. The performance of the new models is compared with existent methods. A section of final remarks concludes the paper.

## 2. Formulations

### 2.1. Introduction

The terminology presented in this section reflects the fact that our study is motivated by a refuse collection problem. The problem undertaken is to plan the collection of garbage in a city with minimum total cost. The street network is described by a mixed graph. Edges characterize two way streets where zig-zag collection is allowed, i.e., the vehicle can collect the garbage in both sides of the street with a single traversal. Arcs represent either one way streets or large two way streets with no zig-zag collection. In the later case one arc for each direction should be included in the network. Nodes characterize the street crossings or dead-end streets. A special node, called *depot*, is the starting and ending point for the vehicle trips. The depot is also the dumpsite, where vehicles empty the refuse collected. A *vehicle trip* is a circuit that can be performed by a vehicle from and back to the depot while servicing the streets (network links), compatible with its capacity. The streets to be served, where there is refuse to be collected, are the *required links* or *tasks*. Some of the streets do not have refuse to be collected and they may be traversed only to ensure the connectivity of the trips. Every street (task or not) traversed by a vehicle without serving it is a *deadheading link*. For simplicity, it is assumed that each vehicle performs only one trip. Capacity and number of vehicles, demands on each street, service and deadheading costs and dump cost at depot are known.

Consider, then, the following notation:

- $\Gamma = (N, A' \cup E)$ is the mixed graph, with $A_R \subseteq A'$ and $E_R \subseteq E$ the set of required arcs and edges, respectively; and $N$ the set of nodes, representing street crossings, dead-end streets, or the depot.
- $0 \in N$ is the depot node where every vehicle trip must start and end ($|N| = n+1$).
- $G = (N, A)$ is a directed graph where each edge from $E$ is replaced by two opposite arcs, i.e., $A = A' \cup \{(i,j),(j,i) : (i,j) \in E\}$.
- $R \subseteq A$ is the set of required arcs in $G$, also named as tasks ($|R| = |A_R| + 2\,|E_R|$).
- $P$ is the maximum number of trips allowed.
- $W$ is the capacity of each vehicle.
- $\lambda$ is the dump cost, paid every time a vehicle is emptied at the depot.
- $d_{ij}$ is the deadheading cost of arc $(i,j) \in A$.
- $c_{ij}$ is the service cost of arc $(i,j) \in R$.
- $q_{ij}$ is the demand of arc $(i,j) \in R$.
- $Q_T$ is the total demand, computed as $Q_T = \sum_{(i,j) \in A_R \cup E_R} q_{ij}$.

The problem is to find a set of no more than $P$ vehicle trips, satisfying the vehicles capacity, starting and ending at the depot, node 0, and servicing all the tasks at minimum total cost.

In the sequel **LF** denotes the linear programming relaxation of formulation **F** and $z_F^*$ the optimal value of **F**.