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MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition

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ABSTRACT

In this paper, we study the unsteady flow and heat transfer of a dusty fluid between two parallel plates with variable viscosity and electric conductivity. The fluid is driven by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates with a Navier slip boundary condition. The governing nonlinear partial differential equations are solved numerically using a semi-implicit finite difference scheme. The effect of the wall slip parameter, viscosity and electric conductivity variation and the uniform magnetic field on the velocity and temperature fields for both the fluid and dust particles is discussed.

1. Introduction

Studies related to flow and heat transfer of dusty fluids in a channel with wall slip are extremely useful in improving the design and operation of many industrial and engineering devices. It has important applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplet sprays [1–6]. The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field is also encountered in a variety of applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters. In these devices, the solid particles in the form of ash or soot are suspended in the conducting fluid as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. The resultant effects of the presence of solid particles on the performance of such devices has led to studies of particulate suspensions in conducting fluids in the presence of an externally applied magnetic field [7–9].

For the MHD flow of incompressible Newtonian fluids, the governing field equations are the incompressible continuity equation, the Navier–Stokes equations and the energy equation. In addition, a boundary condition has to be imposed on the field equations. Traditionally the so-called no-slip boundary condition is used, namely the fluid velocity relative to the solid is zero on the fluid–solid interface [10]. However, the no-slip condition is a hypothesis rather than a condition deduced from any principle, and thus its validity has been continuously debated in the scientific literature. Although many experimental results were shown to support the no-slip condition including those by Coulomb and Couette, evidences of slip of a fluid on a solid surface were also reported by many others [11]. To describe the slip characteristics of fluid on the solid surface, Navier introduced a more general boundary condition, namely the fluid velocity component tangential to the solid surface, relative to the solid surface, is proportional to the shear stress on the fluid–solid interface. The proportionality is called the slip length which describes the "slipperiness" of the surface [12]. Although Navier's slip condition was proposed about two hundred

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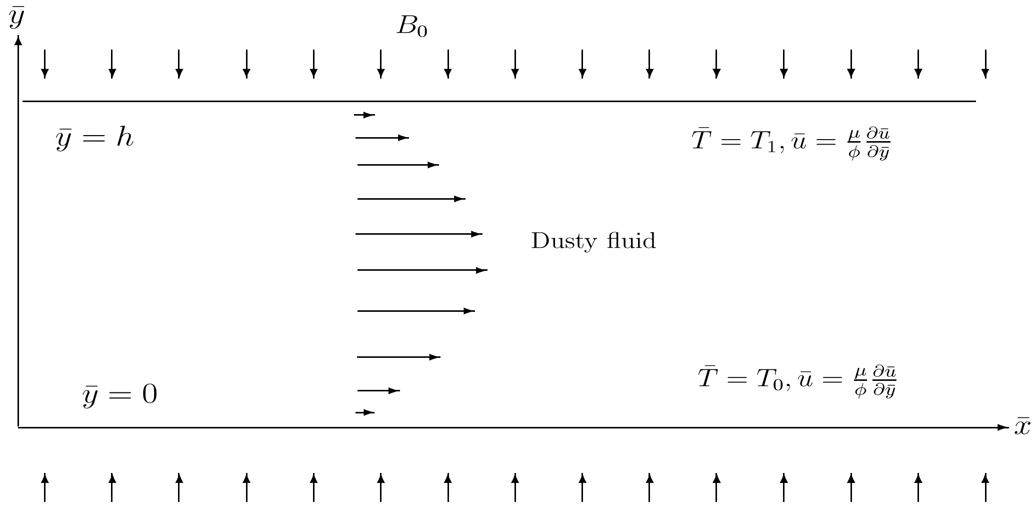

Fig. 1. Schematic diagram of the problem.

years ago, it attracted significant attention of the scientific and engineering communities only very recently for the study of flows on the micro-scale. Recent advances in the manufacture of micro-devices enable experimental investigation of fluid flow on the micro-scale, and many experimental results have provided evidence to support the Navier slip condition [13]. Some attempts have also been made to use nano-technologies for the surface treatment of micro-channels so as to achieve large slip for maximizing the transport efficiency of fluids through them. Over the last few years, various investigations have also been made to study various flow problems of Newtonian and non-Newtonian fluids with or without the Navier slip boundary condition [14–16]. Moreover, accurate prediction for the flow and heat transfer can be achieved by taking into account the variation of these properties with temperature [17]. Attia [8] studied the unsteady MHD flow and heat transfer between two parallel plates with temperature dependent viscosity and thermal conductivity.

In the present work, the unsteady flow and heat transfer of an electrically conducting, incompressible dusty fluid with temperature dependent viscosity and thermal conductivity are studied. The fluid flows between two electrically insulated infinite plates maintained at two constant but different temperatures with the Navier wall slip condition. The fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The dusty-fluid equations discussed by [1–8] are employed, both the fluid and dust particles are governed by the coupled set of the momentum and energy equations. The Joule effect and viscous dissipation are taken into consideration in the energy equation. The governing coupled nonlinear partial differential equations are solved numerically using semi-implicit finite difference methods. The effects of the external uniform magnetic field, wall slip parameter and the variable viscosity and thermal conductivity on the time development of the velocity and temperature distributions for both the fluid and dust particles are discussed.

2. Mathematical model

We consider an unsteady flow of a dusty conducting fluid between two infinite horizontal plates located at the $y = 0, h$ planes. The dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid. The two plates are assumed to be kept at two constant temperatures T_0 for the lower plate and T_1 for the upper plate with $T_1 > T_0$, (see Fig. 1).

A constant pressure gradient is applied in the x-direction and a uniform magnetic field B_0 is applied in the positive y-direction. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected [9]. The fluid motion starts from rest at $t = 0$, and the Navier slip condition at the plates implies that the fluid axial velocity component at the plate surface is proportional to the shear stress on the fluid–plate interface. The initial temperature of both the fluid and dust particles is assumed to be equal to T_0. Both the fluid viscosity and the electric conductivity are assumed to vary with temperature. The particle phase is assumed to be stress free, such as in dilute suspensions, and the hydrodynamic interactions between the phases are limited to the drag force. Other interactions such as the virtual mass force, the shear force, and the spin-lift force are assumed to be negligible compared to the drag force [2–4]. This assumption is feasible when the particle Reynolds number is assumed to be small. Following [1–8], the simplified dimensionless governing equations of momentum and energy balance, together with their corresponding boundary conditions, are given as:

$$\frac{\partial u}{\partial t} = \alpha + \frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) - \frac{\mathrm{Ha}^2}{\mathrm{Re}}u - \frac{R}{\mathrm{Re}}(u - u_p), \tag{1}$$