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Jerks and conductivity anisotropy of lower mantle

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Abstract

Conductivity anisotropy of the lower mantle presumably caused by phase change of dielectric magnesiowüstite at depths of 1500–2000 km is detectable from jerks. Jerks are induced by currents in the fluid outer core, propagate upward from the CMB through anisotropic conducting mantle, and appear on the Earth's surface. The surface jerk patterns are studied theoretically from the potential of the geomagnetic field presented as a sum of magnetic and electric modes. Equations for the fields of both modes and their relationship in a weakly anisotropic earth are obtained by the perturbation method. The field potential is expanded into a series of spherical harmonics, and the equations are solved in the frequency and time domains. The surface jerk responses can be inverted to retrieve anisotropy parameters; the goal function in the inversion may correspond to misfit between the model and experimental values either along the horizontal or vertical components.

Keywords: magnesiowüstite; conductivity anisotropy; global electromagnetic induction; geomagnetic variations; jerks; lower mantle; outer fluid core; core–mantle boundary

Introduction

Geomagnetic jerks or relatively sudden changes in the time derivative trend of geomagnetic secular variations have implications for processes at the core-mantle boundary and, specifically, for lower mantle conductivity (Alexandrescu et al., 1999; Ballani et al., 2002; Nagao et al., 2003; Pinheiro and Jackson, 2008).

Conductivity modeling for depths between 800 and 2900 km from laboratory experiments with minerals at lower mantle temperatures and pressures (Xu et al., 2000) showed good agreement with data on period dependence of apparent resistivity when the model assumes a ~5×10^5 S/m core and a contribution of magnesiowüstite below 800 km. Insulator-to-metal transition of magnesiowüstite was predicted by *P-T* modeling (Ovchinnikov, 2011; Ovchinnikov et al., 2012), and its possible effect was simulated (Plotkin et al., 2013) using spherical harmonic analysis (SHA), at periods from 27 days to several decades. Estimating the predicted effect from apparent resistivity being difficult, frequency dependences of geomagnetic variations at the same data periods were inverted (Plotkin et al., 2014). Inversion of real observatory records (monthly means of the geomagnetic field) likewise indicated possible presence of a conductor in the lower mantle.

The magnesiowüstite phase change in the lower mantle is associated with physical effects in the lattice, and thus may be evidence of conductivity anisotropy. Surface EM responses of an earth with conductivity anisotropy in a mantle spherical layer comprise the magnetic (TE) and electric (TM) modes (Plotkin, 2014). The two modes characterized either jointly or separately provide information on the conductivity tensor of the anisotropic layer. The conductivity tensor components corresponding to the tangential field components have been estimated from *X*, *Y*, and *Z* surface records of the magnetic mode alone. For this we (Plotkin et al., 2015) processed geomagnetic monthly means, for the time span from 1920 to 2009, borrowed from the World Monthly Means Database available as open files. Anisotropy became notable at decadal or longer periods, and its contribution increased with the period length.

The cited studies use data on externally controlled geomagnetic variations. It is interesting to see whether the lower mantle conductivity anisotropy is detectable from effects of

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internal sources, such as jerks. The patterns of jerks in an anisotropic conducting earth are studied theoretically below.

Equations for jerk EM fields

The surface EM field corresponding to a response of an earth with conductivity anisotropy in a spherical mantle layer is presented as a sum of the magnetic and electric modes (Plotkin, 2014).

$$F_\theta = \frac{1}{r}\frac{\partial F^{(1)}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F^{(0)}}{\partial\varphi},\quad F_\varphi = \frac{1}{r\sin\theta}\frac{\partial F^{(1)}}{\partial\varphi} - \frac{1}{r}\frac{\partial F^{(0)}}{\partial\theta}, \quad (1)$$

where the vector $\mathbf{F}(\mathbf{r})$ can stand for any of three vectors: the electric field $\mathbf{E}(\mathbf{r})$, the magnetic field $\mathbf{H}(\mathbf{r})$, and the current $\mathbf{J} = \hat{\sigma}\mathbf{E}$ induced in an anisotropic conducting mantle with the electrical conductivity tensor $\hat{\sigma}$. In spherical coordinates with the origin at the Earth's center, the potentials of the modes $F^{(1)}$ and $F^{(0)}$ are related with the angular distributions of field tangential components on the sphere of any radius as

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(F_\varphi) = \frac{1}{r^2}\Delta_\Omega F^{(1)},$$
$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\varphi) - \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(F_\theta) = -\frac{1}{r^2}\Delta_\Omega F^{(0)}. \quad (2)$$

These potentials are convenient to use for our purposes because (Plotkin, 2014)

$$(\mathrm{rot}\,\mathbf{F})_r = -\frac{1}{r^2}\Delta_\Omega F^{(0)},$$
$$\Delta_\Omega = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} \quad (3)$$
$$(\mathrm{rot\,rot}\,\mathbf{F})_r = \frac{1}{r^2}\Delta_\Omega\left(\frac{\partial F^{(1)}}{\partial r} - F_r\right),$$
$$(\mathrm{rot\,rot\,rot}\,\mathbf{F})_r = \frac{1}{r^2}\Delta_\Omega\left(\frac{\partial^2 F^{(0)}}{\partial r^2} + \frac{1}{r^2}\Delta_\Omega F^{(0)}\right).$$

According to (3), the two modes are independent in spherically symmetrical and isotropic media but are related in the case of anisotropy and deviation from spherical symmetry. Using (3) and Maxwell's equations, their relation can be written as

$$\frac{1}{r^2}\Delta_\Omega\left[\frac{\partial^2 E^{(0)}}{\partial r^2} + \frac{1}{r^2}\Delta_\Omega E^{(0)}\right] - \mu_0\frac{1}{r^2}\Delta_\Omega\frac{\partial J^{(0)}}{\partial t} = 0, \quad (4)$$

$$\frac{1}{r^2}\Delta_\Omega\left(\frac{\partial E^{(1)}}{\partial r} - E_r\right) + \mu_0\frac{\partial J_r}{\partial t} = 0,$$
$$\mathrm{div}\,\mathbf{J} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 J_r) + \frac{1}{r^2}\Delta_\Omega J^{(1)} = 0. \quad (5)$$

for the magnetic (4) and electric (5) modes, respectively. The modes are related in (4) and (5) via the properties of the current potentials. Namely, by substituting $\mathbf{J} = \hat{\sigma}\mathbf{E}$ into (2), taking into account (1), one obtains the potential $J^{(0)}$ from the second equation in (2), in the case of anisotropy, as

$$\frac{1}{r^2}\Delta_\Omega J^{(0)} = \frac{1}{r^2}\left[\frac{\sigma_{\varphi\varphi}}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial E^{(0)}}{\partial\theta}\right) + \frac{\sigma_{\theta\theta}}{\sin^2\theta}\frac{\partial^2 E^{(0)}}{\partial\varphi^2}\right]$$
$$-\frac{\sigma_{\theta\varphi}+\sigma_{\varphi\theta}}{r^2\sin\theta}\frac{\partial^2 E^{(0)}}{\partial\theta\,\partial\varphi} + \frac{1}{r^2}\left[\frac{\sigma_{\theta\varphi}}{\sin^2\theta}\frac{\partial^2 E^{(1)}}{\partial\varphi^2}\right.$$
$$\left.-\frac{\sigma_{\varphi\theta}}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial E^{(1)}}{\partial\theta}\right)\right] + \frac{\sigma_{\theta\theta}-\sigma_{\varphi\varphi}}{r^2\sin\theta}\frac{\partial^2 E^{(1)}}{\partial\theta\,\partial\varphi}$$
$$+\frac{1}{r\sin\theta}\left[\sigma_{\theta r}\frac{\partial E_r}{\partial\varphi} - \sigma_{\varphi r}\frac{\partial}{\partial\theta}(\sin\theta\ E_r)\right]. \quad (6)$$

It is assumed for simplicity that all $\hat{\sigma}$ components are invariable laterally (or with angle).

Jerks are induced by currents in the fluid outer core. The outer core is much more conductive (5×10^5 S/m) than the lower mantle (Xu et al., 2000), and the electric mode induced inside the core can be assumed to remain within the mantle. Then, the external currents $\mathbf{j}_c$ that induce the magnetic field in the mantle form a toroidal system at the core–mantle boundary (CMB), at $r = R_c$ (see (Parkinson, 1983) for more details of poloidal and toroidal fields):

$$\mathrm{div}_\Omega\,\mathbf{j}_c = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta j_{c\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(j_{c\varphi}) = 0,$$
$$j_{c\theta} = \frac{1}{r\sin\theta}\frac{\partial j_c^{(0)}}{\partial\varphi},\quad j_{c\varphi} = -\frac{1}{r}\frac{\partial j_c^{(0)}}{\partial\theta}. \quad (7)$$

With these assumptions, the CMB boundary conditions are

$$H_\theta(R_c,\theta,\varphi,t) = j_{c\varphi}(R_c,\theta,\varphi,t),$$
$$H_\varphi(R_c,\theta,\varphi,t) = -j_{c\theta}(R_c,\theta,\varphi,t). \quad (8)$$

It follows from (7), (1) for the magnetic field, and (8) that

$$H^{(1)}(R_c,\theta,\varphi,t) = -j_c^{(0)}(R_c,\theta,\varphi,t),\ H^{(0)}(R_c,\theta,\varphi,t) = 0. \quad (9)$$

Now it is pertinent to take into account the relation between the magnetic and electric field potentials (Plotkin, 2014):

$$-\frac{1}{r^2}\Delta_\Omega H^{(0)} = J_r,\quad E_r - \frac{\partial E^{(1)}}{\partial r} = -\mu_0\frac{\partial H^{(0)}}{\partial t},$$
$$\frac{1}{r^2}\Delta_\Omega E^{(0)} = \mu_0\frac{\partial H_r}{\partial t},\quad \frac{\partial E^{(0)}}{\partial r} = -\mu_0\frac{\partial H^{(1)}}{\partial t}. \quad (10)$$

The radial current at the CMB being zero, the first equation in (10) again leads to the second condition in (9). With the first and last equations in (9) and (10), the CMB boundary condition for (4) finally becomes

$$\frac{\partial E^{(0)}}{\partial r}(R_c,\theta,\varphi,t) = \mu_0\frac{\partial j_c^{(0)}(R_c,\theta,\varphi,t)}{\partial t}. \quad (11)$$

Complete problem formulation for jerk generation requires one more boundary condition for (4). On the Earth's surface $r = R$, this can be, for instance, one of the two conditions