Available online at www.sciencedirect.com
ScienceDirect
Russian Geology and Geophysics 55 (2014) 89–97
RUSSIAN GEOLOGY AND GEOPHYSICS
www.elsevier.com/locate/rgg

Approximation of a near-vertical boundary in the problems of pulsed electromagnetic soundings

N.V. Shtabel' [a,*], M.I. Epov [a,b], E.Yu. Antonov [a], M.A. Korsakov [a]

[a] A.A. Trofimuk Institute of Petroleum Geology and Geophysics, Siberian Branch of the Russian Academy of Sciences, pr. Akademika Koptyuga 3, Novosibirsk, 630090, Russia
[b] Novosibirsk State University, ul. Pirogova 2, Novosibirsk, 630090, Russia

Received 19 December 2012; accepted 23 May 2013

Abstract

We analyze the results of a mathematical simulation of pulsed electromagnetic fields in geologic media with dipping near-vertical boundaries as well as interpretations within approximating block models and a layered homogeneous conducting model. We consider the possibilities and limitations of these approaches to the inversion of data from pulsed soundings of actual geologic media.

Keywords: vector finite-element method; 3D modeling; pulsed electromagnetic soundings

Introduction and problem formulation

The theory of nonstationary soundings is based on analysis of the spatial distribution of the electromagnetic field in horizontally layered models for conducting nonmagnetic isotropic geologic media. The most widespread ground unit consists of several closed circuits (coils): one transmitter and receivers.

When current within the circuit is switched off, ring eddy current develops beneath the surface of the conducting half-space, according to Faraday's induction law. The electromagnetic field is formed by secondary eddy currents induced in the conducting parts of the medium.

Eddy currents make up moving toroid structures, often called "smoke rings." Some time after the switch-off, the eddy current toroid dips into the medium and its radius and cross-section increase. In the conducting half-space, the toroid center moves along a straight path inclined at ~28° to its surface (Epov et al., 1994; Nabighian, 1979). If a horizontally layered model is based on an insulator, its path levels out and it begins to move horizontally with time (so-called S zone). In this case, the electric field does not cross the inner boundaries, at which conductivity changes in a saltatory manner.

The described process degenerate in terms of the generation of the secondary electromagnetic field, because sources of the second type (charges) do not participate therein. They appear in the medium in the presence of nonhorizontal boundaries. The eddy electric field crosses the inner boundaries, on whose surface charges appear with a density proportional to the average normal component of the electric field at the boundary. The proportionality coefficient is called "the contrast coefficient" and described by the simple expression

$$k_{12} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-},$$

where σ_+, σ_- are the conductivities of the medium at both sides of the boundary.

Importantly, the surface charges are confined to the boundaries and can move only along them. On the other hand, the eddy current toroid moves within the entire half-space over time. In this case, the eddy currents and surface charges interact, especially if there are several dipping boundaries in the medium. The conventional division of the field into two modes—induction (eddy currents) and galvanic (charges)—loses its physical meaning and cannot be used to the full when the behavior of the nonstationary electromagnetic field is analyzed.

Present-day systems of 3D inversion are usually based on a class of models consisting of a set of conducting areas, which are separated by a system of horizontal and vertical bounda-

* Corresponding author.
E-mail address: nadino2000@mail.ru (N.V. Shtabel')

1068-7971/$ - see front matter
http://dx.doi.org/10.1016/j.rgg.2013.12.007

ries. Such a description of the medium leaves open the question of measured-signal inversion in widespread cases, when dipping boundaries are present. How adequate the approximation of the dipping boundary by a set of horizontal and vertical planes is, remains unclear. If their number is large, it is evident from physical considerations that the values of nonstationary electromagnetic fields in these two models will approach one another. If the number of the approximating horizontal and vertical plane boundaries is small, the distribution of surface charges at the dipping boundary will differ considerably. It is continuous at the dipping boundary, and the density at the horizontal boundaries will change in a saltatory manner.

Forward mathematical modeling is used in this study to assess the possibility of describing dipping boundaries by simpler models with horizontal and vertical boundaries.

For simplicity, we consider a model with one dipping (40º) plane separating two areas of different conductivities. It will be approximated by a stepwise set of vertical and horizontal surfaces. The number and sizes of the steps necessary for the approximation of the dipping boundary will be defined so that the nonstationary fields on the surface of the half-space will differ by no more than 1–3%. The behavior of the nonstationary electromagnetic field will be analyzed in more complicated models with several dipping boundaries. Finally, the consequences of model inconsistency in the inversion of such signals within horizontally layered models will be considered.

Models for the medium and sounding unit

The model is a parallelepiped measuring $3 \times 3 \times 4$ km and divided into two subareas: almost nonconducting (air) and conducting (ground), each 2 km high. We presume that the medium is nonmagnetic and nonpolarizing ($\mu = \mu_0 = 4\pi \times 10^7$ H/m, $\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m). The air resistivity is taken equal to 10^6 Ohm·m. The sounding unit, located on the surface of the conducting medium, consists of coaxial square coils (transmitter, 100×100 m; receiver, 25×25 m).

Cartesian coordinates *xyz* are introduced. Their origin coincides with the center of the transmitter coil. The day surface is described by the equation $z = 0$. The inner boundaries are tilted with respect to the day surface. The tilt angles of the boundaries on the vertical plane $x0z$ will be designated by θ.

Model 1 (Fig 1*a*). The conducting area is divided by a dipping plane crossing the day surface at a distance of 500 m from the unit center at θ = 40º. The subarea to the right of the dipping boundary has a resistivity of 200 Ohm·m, whereas that to the left, 10 Ohm·m.

Models 2 and 3 (Fig. 1*b*, *c*) have the same structure but different resistivities. Model 2 has two dipping boundaries. The main boundary *1* is tilted at $\theta = 40°$ (as in model 1) and perpendicular to the additional boundary *2*. The intersection of the dipping boundaries in model 3 is localized 500 m to depth from the unit center. The depth of the intersection of the dipping planes in model 4 is 100 m. The resistivities of the subareas to the right and to the left of boundary *1* are 200 and 10 Ohm·m. The area bounded by the dipping planes in models 2 and 3 might be conducting (resistivity 5 Ohm·m) or nonconducting (resistivity 1000 Ohm·m).

Model 4 (Fig. 1*d*). The conducting area is divided into layers by three horizontal boundaries ($z = 250$, 500, and 750 m). The dipping boundary crosses them at $\theta = 40º$. The upper layer (250 m thick) to the right of the dipping boundary is characterized by two resistivity values: 5 and 1000 Ohm·m. The second layer has a resistivity of 200 Ohm·m, and the underlying layer, 10 Ohm·m.

The modeled signal is nonstationary electromotive force (EMF) observed in the receiver coil after switching off the generator. Recording time, up to 10 ms.

Mathematical simulation

As the studied model is essentially three-dimensional, the simulation method should be selected with regard to the dimensions and configuration of the sounding unit as well as the structure of the area. The method should take into account the influence of boundaries with very contrasting conductivities. The above requirements are fulfilled by the vector finite-element method (VFEM). Tetrahedral elements are used in the triangulation, because they permit "condensing" the grid near the sounding unit (coil–coil) and/or other small elements of the medium. The solution of the forward problem is the EMF induced in the receiver circuit depending on time after switching off the current pulse in the transmitter coil.

The electric field in the quasi-stationary approximation, which appears in the medium after switching off the transmitter current, is described by the corollary of Maxwell's equations

$$\begin{aligned} &\operatorname{rot} \mu^{-1} \operatorname{rot} \mathbf{E} + \sigma \frac{\partial \mathbf{E}}{\partial t} = -\frac{\partial \mathbf{J}^0}{\partial t}, \\ &\mathbf{E} \times \mathbf{n}\big|_{\partial\Omega} = 0, \quad \mathbf{E}\big|_{t=0} = 0, \end{aligned} \tag{1}$$

where $\mathbf{E}$ is the vector of the electric-field intensity; $\mathbf{J}^0$, vector of the extrinsic-current density in the transmitter vs. time; μ, magnetic permeability; σ, conductivity of the medium; and $\partial\Omega$, outer boundaries.

Let us introduce the functional spaces in which the solution of (1) will be searched for:

$$H(\text{rot}, \Omega) = \left\{ \mathbf{u} \in L^2(\Omega) \mid \nabla \times \mathbf{u} \in L^2(\Omega) \right\},$$

$$H_0(\text{rot}, \Omega) = \left\{ \mathbf{u} \in H(\text{rot}, \Omega) \mid \mathbf{u} \times \mathbf{n}|_{\partial\Omega} = 0 \right\}.$$

With the corresponding scalar product and norms,

$$(\mathbf{u}, \mathbf{v})_\Omega = \int_\Omega \mathbf{u} \cdot \mathbf{v}\, dx, \quad \|\mathbf{u}\| = \sqrt{(\mathbf{u}, \mathbf{u})_\Omega},$$

$$\|\mathbf{u}\|^2_{H(\text{rot}, \Omega)} = \|\mathbf{u}\|^2_\Omega + \|\nabla \times \mathbf{u}\|^2_\Omega .$$