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Geomagnetic induction responses of anisotropic conducting mantle

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Abstract

Phase change of dielectric magnesiowüstite in the lower mantle may leave signatures in geomagnetic records of the globally distributed array of observatories. The related features appear in EM induction responses of lower mantle, which are studied theoretically. The surface EM field corresponding to a response of the earth with conductivity anisotropy in a mantle spherical layer is presented as the sum of the magnetic and electric modes. Equations for the fields of both modes and their relationship in a weakly anisotropic earth are obtained by the perturbation method. The two field modes are analyzed jointly and separately to characterize the conductivity tensor of the anisotropic lower mantle. The tensor elements corresponding to the tangential components of the field can be estimated from the magnetic mode alone recorded currently by the global network of geomagnetic observatories. For the tensor data to be complete, observatory data on lateral variations of the electric field are required in addition to three-component geomagnetic records.

Keywords: magnesiowüstite; lower mantle; conductivity anisotropy; induction; three-component records of geomagnetic variations; observatories; horizontal components of electric field

Introduction

The electrical conductivity patterns of the deep earth through ~2000–2500 km can be inferred from global electromagnetic induction data (Rotanova and Pushkov, 1982). Magnetic bays (several hours long) and diurnal variations can record conductivities to depths of ~500–700 km. Longer-period variations from 30 days to one year (Olsen, 1999) have implications for conductivities till ~2000 km, which grow with depth to ~2 S/m at ~800 km and then remain almost invariable in the range 3–10 S/m between 800 and 2000 km. Some information can be also drawn from secular geomagnetic variations and jerks due to internal magnetic sources (Ducruix et al., 1980). CHAMP satellite data on eleven magnetic storms (Velymsky et al., 2006) recorded lower mantle conductivity values of a few S/m. However, the controls and magnitude of the lower mantle electrical conductivity remain poorly constrained. The available geophysical estimates of its magnitude to ~3000 km depths vary from σ~1–3 S/m to more than 10 S/m (Honkura and Matsushima, 1998).

Conductivity modeling for depths between 200 and 2900 km from laboratory experiments with minerals at lower mantle temperatures and pressures (Xu et al., 2000) showed good agreement with observed data in period dependence of apparent resistivity when the model assumed a $\sim 5 \times 10^5$ S/m core and a contribution of magnesiowüstite below 800 km.

Phase changes of minerals at high pressures and temperatures have been a subject of much recent research. Ovchinnikov (2011) predicted possible insulator-to-metal transition of magnesiowüstite at ~60–80 GPa and ~1900–2100 ℃ and then estimated the ensuing conductivity increase at the depths 1400–1900 km (Ovchinnikov et al., 2012). Earlier we (Plotkin et al., 2013) simulated the effect of magnesiowüstite metallization using spherical harmonic analysis of mantle EM responses in observatory geomagnetic records at periods from 27 days to several decades. Estimating the predicted effect from apparent resistivity being difficult, we inverted frequency dependences of geomagnetic variations at the same periods (Plotkin et al., 2014). The inversion procedure was tested in numerical experiments with synthetic geomagnetic data at periods from 50 days to 33 years, and the inversion quality was satisfactory for several lower-order spherical harmonics at known boundaries of spherical layers. Inversion of real observatory records (monthly means of the geomagnetic field from 1920 through 2009) indicates possible presence of a conductor in the lower mantle.

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On the other hand, the magnesiowüstite phase change in the lower mantle is associated with physical effects in the lattice, and thus may be evidence of conductivity anisotropy. It is thus interesting to see whether the lower mantle conductivity anisotropy is detectable by geophysical methods. This is the objective of the reported study of geomagnetic induction responses.

Equations of EM induction in the case of mantle conductivity anisotropy

It is convenient to present the electromagnetic field in the induction problem as a sum of the electric and magnetic modes (Plotkin, 2004). The two modes are independent in spherically symmetrical and isotropic media but are related otherwise. In order to take into account the relationship of the two modes in the case of anisotropy and deviation from spherical symmetry, the tangential components of the electric $\mathbf{E}(\mathbf{r})$ and magnetic $\mathbf{H}(\mathbf{r})$ fields, as well as the current $\mathbf{J}=\hat{\sigma}\mathbf{E}$ induced in an anisotropic earth with the conductivity tensor $\hat{\sigma}$ are written as:

$$E_\theta = \frac{1}{r}\frac{\partial E^{(1)}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial E^{(0)}}{\partial\varphi},\; E_\varphi = \frac{1}{r\sin\theta}\frac{\partial E^{(1)}}{\partial\varphi} - \frac{1}{r}\frac{\partial E^{(0)}}{\partial\theta},$$

$$H_\theta = \frac{1}{r}\frac{\partial H^{(1)}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial H^{(0)}}{\partial\varphi},$$

$$H_\varphi = \frac{1}{r\sin\theta}\frac{\partial H^{(1)}}{\partial\varphi} - \frac{1}{r}\frac{\partial H^{(0)}}{\partial\theta}, \tag{1}$$

$$J_\theta = \frac{1}{r}\frac{\partial J^{(1)}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial J^{(0)}}{\partial\varphi},\; J_\varphi = \frac{1}{r\sin\theta}\frac{\partial J^{(1)}}{\partial\varphi} - \frac{1}{r}\frac{\partial J^{(0)}}{\partial\theta},$$

where r, θ, φ are the spherical coordinates of a point in a system originated at the Earth's center, $E^{(1)}$ and $H^{(0)}$ are, respectively, the scalar potentials of the electric and magnetic fields, and $J^{(1)}$ is the current in the electric mode; $E^{(0)}$, $H^{(1)}$, and $J^{(0)}$ are, respectively, the same parameters for the magnetic mode. As one can check with direct calculations, the two modes can remain independent at this presentation after applying the rot operators (Plotkin, 2005). For example, for the electric field components:

$$(\mathrm{rot}\,\mathbf{E})_r = -\frac{1}{r^2}\Delta_\Omega E^{(0)},\; \Delta_\Omega = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2},$$

$$(\mathrm{rot\,rot}\,\mathbf{E})_r = \frac{1}{r^2}\Delta_\Omega\left(\frac{\partial E^{(1)}}{\partial r} - E_r\right), \tag{2}$$

$$(\mathrm{rot\,rot\,rot}\,\mathbf{E})_r = \frac{1}{r^2}\Delta_\Omega\left(\frac{\partial^2 E^{(0)}}{\partial r^2} + \frac{1}{r^2}\Delta_\Omega E^{(0)}\right).$$

Using Maxwell's equations $\mathrm{rot}\,\mathbf{E} = -i\omega\mu_0\mathbf{H}$ and $\mathrm{rot}\,\mathbf{H} = \mathbf{J}$ for the fields $\sim e^{i\omega t}$ (μ_0 and ω are the magnetic permeability and the angular frequency, respectively) and their consequences

$$\mathrm{rot\,rot}\,\mathbf{E} + i\omega\mu_0\mathbf{J} = 0,\; \mathrm{rot\,rot\,rot}\,\mathbf{E} + i\omega\mu_0\,\mathrm{rot}\,\mathbf{J} = 0,$$

it is easy to obtain the system of equations for the potentials of the EM modes at general assumptions on conductivity anisotropy in a heterogeneous earth:

$$\frac{1}{r^2}\Delta_\Omega\left[\frac{\partial^2 E^{(0)}}{\partial r^2} + \frac{1}{r^2}\Delta_\Omega E^{(0)}\right] - i\omega\mu_0\frac{1}{r^2}\Delta_\Omega J^{(0)} = 0, \tag{3}$$

$$\frac{1}{r^2}\Delta_\Omega\left(\frac{\partial E^{(1)}}{\partial r} - E_r\right) + i\omega\mu_0 J_r = 0,$$

$$\mathrm{div}\,\mathbf{J} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 J_r) + \frac{1}{r^2}\Delta_\Omega J^{(1)} = 0. \tag{4}$$

The potentials of the magnetic and electric modes are related as (Plotkin, 2004):

$$-\frac{1}{r^2}\Delta_\Omega H^{(0)} = J_r,\quad E_r - \frac{\partial E^{(1)}}{\partial r} = -i\omega\mu_0 H^{(0)},$$

$$\frac{1}{r^2}\Delta_\Omega E^{(0)} = i\omega\mu_0 H_r,\quad \frac{\partial E^{(0)}}{\partial r} = -i\omega\mu_0 H^{(1)}. \tag{5}$$

The last equation in system (4) does not include the magnetic mode current potential $J^{(0)}$, which is excluded by the angular term of the div operator, while system (3) does not include the current potential $J^{(1)}$ of the electric mode according to the first equation in (2). Systems (3) and (4) are, respectively, for the magnetic and electric modes, which are related uniquely via the properties of the current potentials. For this case, the following equations are valid, which can be checked by substituting J_θ and J_φ from (1):

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,J_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(J_\varphi) = \frac{1}{r^2}\Delta_\Omega J^{(1)},$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,J_\varphi) - \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(J_\theta) = -\frac{1}{r^2}\Delta_\Omega J^{(0)}. \tag{6}$$

Equations (6) for the tangential components of the current $\mathbf{J}$ (and similar equations for $\mathbf{E}$ and $\mathbf{H}$) allow estimating the potentials of the electric $J^{(1)}$ and magnetic $J^{(0)}$ modes on the sphere of any radius if the angular distributions of the components are known.

The modes and systems (3) and (4) are independent in a spherically layered isotropic earth (since $\mathbf{J} = \sigma(r)\,\mathbf{E}$ while (1) leads to $J^{(0)} = \sigma(r)\,E^{(0)}$ and $J^{(1)} = \sigma(r)\,E^{(1)}$), but are related in a laterally heterogeneous earth (Plotkin, 2004).

In the case of conductivity anisotropy,

$$J_r = \sigma_{rr}E_r + \sigma_{r\theta}E_\theta + \sigma_{r\varphi}E_\varphi,$$

$$J_\theta = \sigma_{\theta r}E_r + \sigma_{\theta\theta}E_\theta + \sigma_{\theta\varphi}E_\varphi, \tag{7}$$

$$J_\varphi = \sigma_{\varphi r}E_r + \sigma_{\varphi\theta}E_\theta + \sigma_{\varphi\varphi}E_\varphi.$$

The main focus in this consideration being on anisotropy, we may assume for simplicity that all components are invariable laterally (or with angle), though may vary with the