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Fast 3D inversion of gravity data using solution space priorconditioned lanczos bidiagonalization

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ABSTRACT

Inversion of gravity data is one of the most important steps in the quantitative interpretation of practical data. Inversion is a mathematical technique that automatically constructs a subsurface geophysical model from measured data, incorporating some priori information. Inversion of gravity data is time consuming because of increase in data and model parameters. Some efforts have been made to deal with this problem, one of them is using fast algorithms for solving system of equations in inverse problem. Lanczos bidiagonalization method is a fast algorithm that works based on Krylov subspace iterations and projection method, but cannot always provide a good basis for a projection method. So in this study, we combined the Krylov method with a regularization method applied to the low-dimensional projected problem. To achieve the goal, the orthonormal basis vectors of the discrete cosine transform (DCT) were used to build the low-dimensional subspace. The forward operator matrix replaced with a matrix of lower dimension, thus, the required memory and running time of the inverse modeling is decreased by using the proposed algorithm. It is shown that this algorithm can be appropriate to solve a Tikhonov cost function for inversion of gravity data. The proposed method has been applied on a noise-corrupted synthetic data and field gravity data (Mobrun gravity data) to demonstrate its reliability for three dimensional (3D) gravity inversion. The obtained results of 3D inversion both synthetic and field gravity data (Mobrun gravity data) indicate the proposed inversion algorithm could produce density models consistent with true structures.

1. Introduction

Gravity measurements have been used in a wide range of investigations including estimation of the crustal thickness, mapping bedrock topography, mineral and petroleum exploration, and recently developed microgravity investigations such as engineering and environmental problems (Hinze, 1990; Ward, 1990; Nabighian et al., 2005). The inversion of gravity data is an important step in the quantitative interpretation of practical data, since construction of density contrast models significantly increases the amount of information that can be achieved from the gravity data (Li and Oldenburg, 1998). Inversion is defined as a mathematical technique that automatically constructs a subsurface physical property model from measured data by incorporating a priori information. The recovered models must predict measured data adequately (Foks et al., 2014). The solution of the inverse problem is dependent upon the formulation and discretization of the geophysical forward problem (Martin et al., 2013). In the linear inversion of gravity data it is assumed that the subsurface under the survey area can be discretized into rectangular blocks of constant density (Boulanger and Chouteau, 2001; Vatankhah et al., 2015). The density of these blocks are approximated by solving the linear inverse problem. Linear inversion of gravity data is usually ill-posed and the solution can be non-unique and unstable. The conventional way of solving ill-posed inverse problems is using regularization theory (Tikhonov et al., 1977). It uses the minimization of a cost function that combines the data misfit with a Tikhonov type regularization (see, e.g. Vogel, 2002; Hansen, 2007; Aster et al., 2013).

Because lack of depth resolution in inversion of gravity data, Li and Oldenburg (1998) introduced a depth-weighting function that counteract the decreasing sensitivities of cells with increasing depth. The depth-weighting gives more weight to cells as depth increases. Depth weighting function has been applied in different inversion algorithms (Boulanger and Chouteau, 2001; Portniaguine and Zhdanov, 2002; Malehmir et al., 2009; Namaki et al., 2011; Vatankhah et al., 2015).

Inversion of gravity data can suffer from large processing times as new sensors and acquisition platforms continue to collect dense data sets over large exploration regions. This problem is compounded by the necessary increase in model parameters (Foks et al., 2014). For

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fast inversion some efforts has been made in some areas: the forward modeling algorithm, matrix storage, the number of model parameters and the number of data, and using a fast and efficient algorithm for solving system of equations like the Lanczos bidiagonalization method.

For fast forward modeling, Pilkington (1997) utilized the 2D fast Fourier transform and convolution theorem while Caratori Tontini et al. (2009) used the 3D fast Fourier transform for the aim. The wavelet transform with orthonormal compactly supported wavelets has been utilized to compress kernel matrix represents the dense matrix in sparse format (Li and Oldenburg, 2003; Davis and Li, 2011; Martin et al., 2013). The number of model parameters has been reduced by mesh refinement (Ascher and Haber, 2001). Quadtree and octree mesh discretization has been utilized in potential field forward modeling and inversion which data adaptively construct a mesh based on resolution of anomaly (Davis and Li, 2011, 2013). Foks et al. (2014) have used an adaptive downsampling method to reduce the number of potential-field data for forward modeling and inversion. Fast solver algorithm like conjugate gradient (CG) has been applied for computation of model parameters (Li and Oldenburg, 2003; Namaki et al., 2011). Recently, Lanczos bidiagonalization (LB) method (Paige and Saunders, 1982) has been utilized for inversion of potential field data that is faster than CG method (Abedi et al., 2013; Martin et al., 2013).

Both CG and LB methods work based on Krylov subspace iterations and projection method. The Krylov subspace cannot always provide a good basis for a projection method, because Krylov subspace is generated on the basis of the given, noisy data. So, the Krylov subspace tends to include both the desired basis vectors and some basis vectors that are not so important for the regularized solution (Hansen, 2010). The solution could be attained faster by removing this problem. Two level methods have been applied for the solution of the Tikhonov problem. In these methods the solution space is divided into two subspaces, one of them with a small dimension that utilizes basis vectors chosen such that this subspace represents approximate regularized solutions with a direct method, while the component of the solution in the remaining subspace is computed by an iterative algorithm (Hanke and Vogel, 1999; Jacobsen et al., 2003). Hansen (2010) proposed an algorithm that combines the Krylov method with a regularization method applied to the low-dimensional projected problem. For fast linear inversion of gravity data, we implemented the solution space priorconditioning with LB method. This paper explains the solution space priorconditioning that the hybrid of cosine transform and Krylov projection algorithm is used for fast linear inversion of gravity data and shows how this approach can reduce required memory and time for inversion of the geophysical data.

2. Methodology

To perform inverse modeling, the subsurface under the survey area is discretized into rectangular prisms of known sizes and positions. The density contrasts within each prism is unknown parameter to be estimated by solving inverse problem. A linear relationship between density and gravity anomaly is a valid approximation; therefore, the inverse solution obtained by solving a linear system of equations (Bear et al., 1995; Martin et al., 2013).

2.1. Forward modeling

Here, the formula given by Blakely (1996) has been used to compute the gravity response of each prism, after discretization of subsurface by rectangular prisms. If the observed gravity anomalies are caused by n subsurface rectangular prisms, the gravity anomaly at the field point i is given by

$$g_i=\sum_{j=1}^{n} G_{ij}\rho_j,\ \ i=1,\ldots,m \tag{1}$$

Where g_i is gravity observation at the point i, ρ_j is density contrast of prism j and G_{ij} relates gravity observation at the point i to the subsurface rectangular prism j with unit density. In the matrix notation Eq. (1) can be written as

$$\mathrm{G}\mathbf{m}=\mathbf{d}, \mathrm{G}\in \mathrm{R}^{\mathrm{m}\times \mathrm{n}}, \mathbf{d}\in \mathrm{R}^{\mathrm{m}}, \mathbf{m}\in \mathrm{R}^{\mathrm{n}} \tag{2}$$

Here, G is forward operator matrix that is also called sensitivity matrix that maps the physical parameters space into the data space. $\mathbf{m}$ denotes the vector of unknown model parameters and $\mathbf{d}$ is data vector that is given by measurements ($\mathrm{g_i}$). There are some error in measurement data because of noise that assumed to be uncorrelated and have Gaussian distribution, So

$$\mathrm{G}\mathbf{m}=\mathbf{d}+\mathbf{e}, \mathbf{e}\in \mathrm{R}^{\mathrm{m}} \tag{3}$$

Where $\mathbf{e}$ is vector of data error and $\mathbf{d}_{\mathrm{obs}}=\mathbf{d}+\mathbf{e}$ is vector of observed data. The main purpose of the gravity inverse problem is to find a geologically plausible density model ($\mathbf{m}$) that predicts measured data ($\mathbf{d}_{\mathrm{obs}}$) at the noise level (Vatankhah et al., 2015).

2.2. Inverse modeling

In order to compute an approximate solution of the density distribution $\mathbf{m}$ in Eq. (3), the inverse problem is formulated by the minimization of the penalized least squares Tikhonov parametric functional as: (Tikhonov et al., 1977)

$$\mathbf{m}(\alpha)=\underset{\mathrm{m}}{\arg\min}\left\{\left\|\mathrm{W_d}(\mathrm{Gm}-\boldsymbol{d}_{\mathrm{obs}})\right\|_2^2+\alpha\left\|\mathrm{Dm}\right\|_2^2\right\} \tag{4}$$

where $\mathbf{m}(\alpha)$ is desired density model, $\Phi(\boldsymbol{d})=\left\|\mathrm{W}_d(Gm-\boldsymbol{d}_{\mathrm{obs}})\right\|_2^2$ is weighted data misfit and $\Phi(\boldsymbol{m})=\left\|\mathrm{Dm}\right\|_2^2$ is the Tikhonov regularization function. W_d is data weighting matrix given by $\mathrm{W}_d=diag(1/\sigma_1,\ldots,1/\sigma_m)$. Where, σ_i stands for the standard deviation of the noise in the ith observation data. D depicts regularization matrix and $\alpha>0$ is regularization parameter or tradeoff parameter which controls relative balance between the data misfit and Tikhonov regularization function.

We applied $\mathrm{W}_{depth}=diag(1/(z_1)^{\beta},\ldots,1/(z_n)^{\beta})$ as a depth weighting matrix (Li and Oldenburg, 1998) to compensate lack of the data sensitivity to the deeper model parameters. Here, z_j is depth of jth model parameter and $\beta=1$ is suitable for inversion of gravity data (Li and Oldenburg, 1998). Depth weighting matrix can be entered into Eq. (4) by replacing regularization matrix (D) with depth weighting matrix (W_{depth}), D= W_{depth}. Introducing, $\mathrm{G}_{\xi}=W_dGD^{-1}$, $\boldsymbol{y}$=Dm and $\boldsymbol{d}_{\xi}=W_d\boldsymbol{d}_{\mathrm{obs}}$, solution of the gravity inversion is equivalent to solving an inverse problem whose forward mapping is given by

$$\mathrm{G}_{\xi}\boldsymbol{y}=\boldsymbol{d}_{\xi} \tag{5}$$

The solution of Eq. (5) is given by

$$\mathbf{y}(\alpha)=\underset{\mathrm{y}}{\arg\min}\left\{\left\|G_{\xi}\boldsymbol{y}-\boldsymbol{d}_{\xi})\right\|_2^2+\alpha\left\|\boldsymbol{y}\right\|_2^2\right\} \tag{6}$$

Depth weighting matrix is diagonal and has an inverse, so $\boldsymbol{m}(\alpha)$ is obtained by

$$\mathbf{m}(\alpha)=\mathrm{D}^{-1}\boldsymbol{y}(\alpha) \tag{7}$$