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Source imaging of potential fields through a matrix space-domain algorithm

Jamaledin Baniamerian [a], Behrooz Oskooi [a,*], Maurizio Fedi [b]

[a] Department of Earth Physics, Institute of Geophysics, University of Tehran, Iran
[b] Department of Earth Sciences, Environment and Resources, University of Naples 'Federico II', Naples, Italy

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ABSTRACT

Imaging of potential fields yields a fast 3D representation of the source distribution of potential fields. Imaging methods are all based on multiscale methods allowing the source parameters of potential fields to be estimated from a simultaneous analysis of the field at various scales or, in other words, at many altitudes. Accuracy in performing upward continuation and differentiation of the field has therefore a key role for this class of methods. We here describe an accurate method for performing upward continuation and vertical differentiation in the space-domain. We perform a direct discretization of the integral equations for upward continuation and Hilbert transform; from these equations we then define matrix operators performing the transformation, which are symmetric (upward continuation) or anti-symmetric (differentiation), respectively. Thanks to these properties, just the first row of the matrices needs to be computed, so to decrease dramatically the computation cost. Our approach allows a simple procedure, with the advantage of not involving large data extension or tapering, as due instead in case of Fourier domain computation. It also allows level-to-drape upward continuation and a stable differentiation at high frequencies; finally, upward continuation and differentiation kernels may be merged into a single kernel. The accuracy of our approach is shown to be important for multi-scale algorithms, such as the continuous wavelet transform or the DEXP (depth from extreme point method), because border errors, which tend to propagate largely at the largest scales, are radically reduced. The application of our algorithm to synthetic and real-case gravity and magnetic data sets confirms the accuracy of our space domain strategy over FFT algorithms and standard convolution procedures.

1. Introduction

Multiscale analysis methods are based on the interpretation of potential fields at different levels and are useful to yield a fast 3D imaging of the source distribution, which may be used as a first stage of interpretation, before performing more refined methods such as inversion. Many multiscale methods have been developed for potential field interpretation. This class of methods is mainly based on upward continuation of potential fields (Pedersen, 1991; Sailhac and Gibert, 2003; Fedi and Florio, 2006; Fedi, 2007; Florio et al., 2009; Fedi et al., 2009, 2012; Fedi and Abbas, 2013; Abbas et al., 2014; Baniamerian et al., 2016). Recently Fedi and Pilkington (2012) have demonstrated that all imaging methods, e.g., Sandwich model (Pedersen, 1991), correlation (Patella, 1997), migration (Zhdanov, 2002), wavelet transform (Moreau et al., 1997) and depth from extreme points (DEXP; Fedi, 2007) are, in practice, multiscale methods and involve upward continuation or a simultaneous application of upward continuation and differentiation to potential fields. Therefore, when using imaging methods it is very important to perform the whole procedure as accurately as possible and, in particular, reduce noise/edge effects as well.

As regards upward continuation, it is well known that, from Green's third identity (e.g., Baranov, 1975; Parasnis, 1986; Blakely, 1996)), a potential field measured on a plane surface at altitude z', $U(x',y',z')$, can be transformed to that at a higher-level surface, by using the upward continuation linear transformation:

$$U(x,y,z) = -\frac{z-z'}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{U(x',y',z')}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}dx'dy', \quad (1)$$

where $\{x,y,z\}$ are the coordinates of any point on a surface characterized by $z>z'$. This transformation behaves like a low-pass filter, since it attenuates the short-wavelength contributions to the signal, and it can be used to enhance the anomalies due to deep sources. However, differently from other low-pass filters, it is expressly related to the physically-based variation of the field with the altitude (Florio et al., 2014). The simplest form of applying Eq. (1) is the level-to-level upward continuation. However, we need often applying upward continuation as a level-

* Corresponding author.
E-mail addresses: baniamerian@ut.ac.ir (J. Baniamerian), boskooi@ut.ac.ir (B. Oskooi), fedi@unina.it (M. Fedi).

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to-draped, draped-to-level or draped-to-draped continuation (Cordell, 1985; Fedi et al., 1999; Xia et al., 1993; Wang, 2006; Pilkington and Roest, 1992; Ridsdill-Smith, 2000; Mastellone et al., 2014).

The other main transformation needed for a multiscale analysis (e.g., Fedi et al., 2009, 2012) is the field differentiation. Field differentiation is widely used to detect the boundary and the horizontal location of sources, to decrease the interference effect of nearby sources, to emphasize the effects from shallow sources or as a necessary tool for powerful interpretative methods (e.g.:, Nabighian, 1972; Thompson, 1982; Reid et al., 1990; Salem and Ravat, 2003; Thurston and Smith, 1997; Fedi and Florio, 2001; Cooper and Cowan, 2003; Fedi et al., 2009).

The easiest way to compute upward continuation and differentiation of potential fields is performing convolution in the frequency domain through conventional FFT algorithms. Convolution is an integral transformation, involving the function U to transform and some filter h performing the transformation itself. Thanks to the convolution theorem (e.g., Bracwell, 2000) convolution is greatly simplified in the frequency domain, since it is reduced to a simple product by the Fourier transforms of U and h. It may be so performed in three steps: Fourier transformation of the potential field; its multiplication by a filter operator; inverse Fourier transformation of such product.

For instance, the level-to-level upward continuation operator in frequency domain is given by (Blakely, 1996):

$$F_{up} = Fe^{-|\mathbf{k}|\Delta z}, \Delta z > 0 \quad (2)$$

where $|\mathbf{k}|$ is the wavenumber modulus: $|\mathbf{k}| = \sqrt{{k_x}^2 + {k_y}^2}$, F is the Fourier transforms of the measured fields U at the altitude z', F_{up} is the Fourier transforms of the upward continued field at the altitude z and Δz is the upward continuation step, $\Delta z = z - z'$. Although the FFT technique is very fast and easy to use, there are some problems with this algorithm. The main problem is the aliasing error due to circular convolution implied in the Fourier domain. It degrades progressively the low-frequency content of upward continued field as the altitude increases (Oppenheim and Schafer, 1975; Fedi et al., 2012). Besides, as the Fourier transform inherently assumes that the function is periodic, if the edges of data are not smooth, the boundaries behave like discontinuities in Fourier transform, and the edge errors are severely intensified (Oppenheim and Schafer, 1975; Blakely, 1996; Mastellone et al., 2014). These errors can be overcome to some extent by computing the upward continuation on a larger area than that of the measurements, or by a windowing technique. If real data are available beyond the interested area, they can be used to the end of enlarging the dataset. Otherwise, the data must be extended to a larger area by mathematical extrapolation methods like zero-padding, smooth extension, symmetric extension or other algorithms. Fedi et al. (2012) gave a comprehensive description on the different kinds of data expansions and on their relative accuracy.

One more critical issue with FFT algorithm is its sensitivity to local high-frequency noise in the dataset. Being the Fourier transformation a global method, low-pass filtering cannot act locally so that a general distortion due to the low-pass filtering is generated through the whole map.

The upward continuation and the differentiation of potential fields can be however directly performed in space domain. Gibert and Galdeano (1985) computed first the filter operators in the frequency domain and then inverse transformed it into the space domain; finally they computed upward continuation and differentiation by convolution in the space domain. Due to Fourier and inverse Fourier transforms during this process, some errors may propagate into the designed convolution filter, which affect the accuracy of the final results. Considering that the computational domain is large, Wang (2006) and Wang et al. (2008) used the spline technique to carry out these operations. In their way, the integrand is approximated by a spline at each point. Here, we solve the problem in the space domain by following a different approach. The integral equations of upward continuation (see Eq. (1)) and vertical differentiation (see Section 1.2) are discretized and written in a matrix equation form. The transformed field, either the upward continued field or the field derivative, is computed through the application of a symmetric or antisymmetric matrix operator. In order to show the performance of the proposed approach, we will consider the DEXP transformation (Fedi, 2007), an imaging method based on upward continuing the field to a set of altitudes; we will compare the DEXP transformed fields generated for synthetic and real data by means of FFT and space-domain algorithms, respectively.

In the following section we will describe our space-domain procedure for upward continuing and differentiating the potential fields.

1.1. Upward continuation of potential fields in space domain

Upward continuation is widely performed in the frequency domain through the fast Fourier transform (FFT) technique in three steps: Fourier transformation of the potential field, its multiplication by a filter operator, and inverse Fourier transformation of such product. Thanks to the Convolution Theorem (e.g., Bracwell, 2000), the level-to-level upward continuation (Eq. (1)) is written in frequency domain very easily as in Eq. (2) and applies to the gravity and magnetic fields and to their components or derivatives of any order.

It is possible to consider the integral Eq. (1), as a linear matrix equation that relates the measured field to the upward continued field

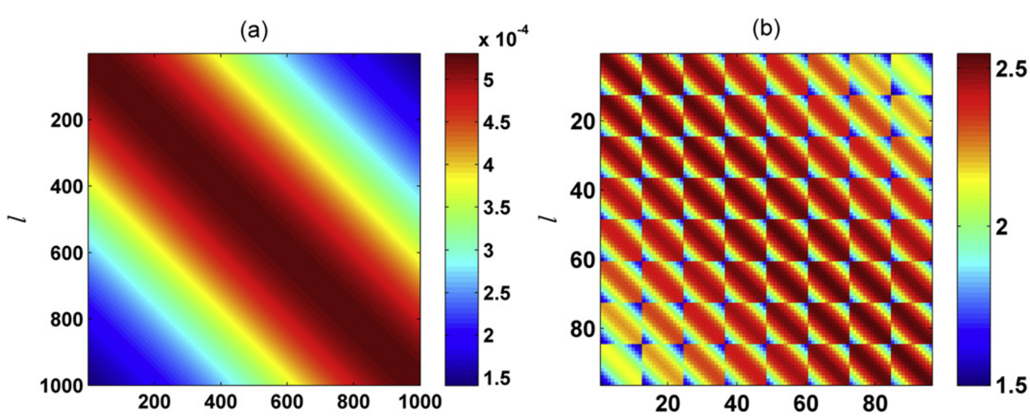

Fig. 1. Typical kernel matrices **K** of upward continuation. (a) the 2D case. There are 1000 points along the profile and level of continuation is 80 m; (b) the 3D case. The level of continuation is 20 m and the cell size is 1 m.