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An approximation to the reflection coefficient of plane longitudinal waves based on the diffusive-viscous wave equation

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ABSTRACT

The frequency-dependent seismic anomalies related to hydrocarbon reservoirs have lately attracted wide interest. The diffusive-viscous model was proposed to explain these anomalies. When an incident diffusive-viscous wave strikes a boundary between two different media, it is reflected and transmitted. The equation for the reflection coefficient is quite complex and laborious, so it does not provide an intuitive understanding of how different amplitude relates to the parameters of the media and how variation of a particular parameter affects the reflection coefficient. In this paper, we firstly derive a two-term (intercept-gradient) and three-term (intercept-gradient-curvature) approximation to the reflection coefficient of the plane diffusive-viscous wave without any assumptions. Then, we study the limitations of the obtained approximations by comparing the approximate value of the reflection coefficient with its exact value. Our results show that the two approximations match well with the exact solutions within the incident angle of 35°. Finally, we analyze the effects of diffusive and viscous attenuation parameters, velocity and density in the diffusive-viscous wave equation on the intercept, gradient and curvature terms in the approximations. The results show that the diffusive attenuation parameter has a big impact on them, while the viscous attenuation parameter is insensitive to them; the velocity and density have a significant influence on the normal reflections and they distinctly affect the intercept, gradient and curvature term at lower acoustic impedance.

1. Introduction

The amplitude variation with offset/angle of incidence (AVO/AVA) has been a powerful technique for geoscientists to extract fluid and lithology information from the analysis of prestack seismic amplitudes. When an incident plane wave is injected on a boundary between two media, it is reflected and transmitted. The theory of solving the problems of reflection and transmission at an interface is Zoeppritz equations in elastic media (Zoeppritz, 1919). Zoeppritz equations give exact values for the amplitudes of the reflected and transmitted plane waves. However, they do not support an intuitive understanding of the effects of the variation of a parameter on the seismic amplitudes. In the past few decades, many linear approximations of Zoeppritz equations have been derived to give an intuitive relationship between parameters of media and seismic amplitudes. The first approximation was obtained by Bortfeld (1961), who linearized the equations by dividing the major subsurface interfaces into a group of layers under the assumptions of small changes in the elastic parameters at the transition layers. This approximation is valid for all precritical angles. Aki and Richards (2002) derived a linearized equation for the reflection compressional wave in such a form that comprises three terms involving density, P-wave velocity and S-wave velocity, for small changes in the P-wave velocity, S-wave velocity and density across a boundary between two elastic media. Wiggins et al. (1983) derived a rearrangement of the Aki-Richards approximation in a three-term form including the intercept, gradient and curvature (intercept-gradient-curvature equation). Shuey (1985) rearranged the Aki-Richards equation by transforming the equation derived by Wiggins et al. (1983) from dependence on V_p, V_s and ρ (the P-wave velocity, S-wave velocity and density, respectively) to dependence on V_p, ρ, and Poisson's ratio ν. Smith and Gidlow (1987) obtained another approximation by using Gardner's velocity-density empirical relationship. Fatti et al. (1994) derived a three-term approximation including P-impedance, S-impedance and density, which is much more extensively applied in field data. Gray et al. (1999) reformulated the Aki and Richards approximation by using two sets of fundamental constants: λ_{sat}, μ_{sat}, and ρ_{sat} (the first and second Lamé parameters and density, respectively), and K_{sat}, μ_{sat}, and ρ_{sat} (bulk modulus, shear modulus, and density, respectively).

More recently, Russell et al. (2011) derived a generalized equation based on Biot-Gassmann poroelasticity theory which contains both of the Gray parameterizations as special cases. Zhao et al. (2014) derived the frequency-dependent reflection coefficient based on the diffusive-

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viscous wave equation proposed by Korneev et al. (2004). However, the reflection coefficient is complicated and does not provide a direct relationship between the variation of a particular parameter in a medium and the reflection coefficient.

In this work, we put forward a method for approximating the reflection coefficient in diffusive-viscous media. We firstly give the exact expression of the frequency-dependent reflection coefficient, and we further derive a two-term (intercept-gradient) approximation and a three-term (intercept-gradient-curvature) approximation to the reflection coefficient of plane diffusive-viscous wave. Then, we investigate the limitations of the approximations by comparing the approximate solutions with the exact solution. Finally, we study the effects of the parameters (diffusive and viscous attenuation parameters, velocity and density) in the diffusive-viscous wave equation on the approximations.

2. The derivation of approximations

In this section, we firstly introduce the diffusive-viscous wave equation, and give the proved exact equation for the frequency-dependent reflection coefficient in the diffusive-viscous media. Then, we derive the approximate equations for the reflection coefficient in order to establish an intuitive relationship between the parameters of the media and the amplitude.

2.1. The frequency-dependent reflection coefficient based on the diffusive-viscous wave equation

The diffusive-viscous wave equation in a 1-D medium is proposed (Goloshubin and Korneev, 2000; Korneev et al., 2004) and mathematically described as follows:

$$\frac{\partial^2 u}{\partial t^2}+\gamma\frac{\partial u}{\partial t}-\eta\frac{\partial^3 u}{\partial x^2\partial t}-\upsilon^2\frac{\partial^2 u}{\partial x^2}=0 \tag{1}$$

where, u is the wave field; γ and η is the diffusive and viscous attenuation parameter respectively, which are functions of porosity and permeability of rocks as well as the viscosity and density of the fluid; υ is the wave propagation velocity in a non-dispersive medium. t is the time and x is the space variable. This equation can be extended to a 2-D case as (He et al., 2008; Zhao et al., 2014)

$$\frac{\partial^2 u}{\partial t^2}+\gamma\frac{\partial u}{\partial t}-\eta\left(\frac{\partial^3 u}{\partial x^2\partial t}+\frac{\partial^3 u}{\partial z^2\partial t}\right)-\upsilon^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)=0 \tag{2}$$

When a plane wave is incident on a planar interface that separates two diffusive-viscous media, the reflection coefficient, R, can be obtained as (Zhao et al., 2014)

$$R=\frac{\rho_2 V_2\cos\theta_1-\rho_1 V_1\cos\theta_2}{\rho_2 V_2\cos\theta_1+\rho_1 V_1\cos\theta_2} \tag{3}$$

where,

$$V_j=\sqrt{\frac{\upsilon_j^2+i\eta_j\omega}{1-i\dfrac{\gamma_j}{\omega}}},\quad j=1,2 \tag{4}$$

here, ρ_1, ρ_2, and V_1, V_2 are the densities and complex velocities of upper medium 1 and bottom medium 2, respectively. θ_1 is the incident angle and θ_2 is the angle of transmission. ω is the angle frequency, and $i=\sqrt{-1}$.

It is found that the Eq. (3) is quite complex and does not provide the intuitive understanding of the effects of variation of parameters in the media on the amplitude. In the following, we derive an approximate equation for Eq. (3).

2.2. The approximations to the frequency-dependent reflection coefficient

In this section, we derive a two-term and three-term approximation to the reflection coefficient R in Eq. (3). Using the Snell's law, Eq. (3) can be rewritten as a function of incident angle θ as

$$R(\theta)=\frac{\rho_2 V_2\sqrt{1-\sin^2\theta}-\rho_1 V_1\sqrt{1-\dfrac{V_2^2}{V_1^2}\sin^2\theta}}{\rho_2 V_2\sqrt{1-\sin^2\theta}+\rho_1 V_1\sqrt{1-\dfrac{V_2^2}{V_1^2}\sin^2\theta}} \tag{5}$$

On the basis of Taylor's series expansion (Ahlfors, 1953), we rewrite the Eq. (5) as

$$R(\theta)\approx R_0+\left(\frac{\partial R}{\partial\sin\theta}_{\sin\theta=0}\right)\sin\theta+\frac{1}{2!}\left(\frac{\partial^2 R}{\partial\sin^2\theta}_{\sin\theta=0}\right)\sin^2\theta+\ldots \tag{6}$$

where,

$$R_0=R(\sin\theta=0)=\frac{\rho_2 V_2-\rho_1 V_1}{\rho_2 V_2+\rho_1 V_1} \tag{7}$$

In Eq. (7), R_0 is the value of reflection coefficient for normal incidence; θ is the incident angle; $\frac{\partial R}{\partial\sin\theta}$ and $\frac{\partial^2 R}{\partial\sin^2\theta}$ are the first-order and the second-order derivative of R with respect to $\sin\theta$, respectively.

It is obviously noted that the values of odd-order derivatives at $\sin\theta=0$ are zeros as $R(\theta)$is an even function with respect to $\sin\theta$. Thus, the first- and third-order derivatives in Eq. (6) are zeros, that is

$$\left.\frac{\partial R}{\partial\sin\theta}\right|_{\sin\theta=0}=0,\quad \left.\frac{\partial^3 R}{\partial\sin^3\theta}\right|_{\sin\theta=0}=0 \tag{8}$$

After some complex algebraic operations, we get

$$\left.\left(\frac{\partial^2 R}{\partial\sin^2\theta}\right)\right|_{\sin\theta=0}=\frac{2\rho_1\rho_2 V_1 V_2\left(\dfrac{V_2^2}{V_1^2}-1\right)}{(\rho_2 V_2+\rho_1 V_1)^2} \tag{9}$$

and

$$\begin{aligned}\left.\left(\frac{\partial^4 R}{\partial\sin^4\theta}\right)\right|_{\sin\theta=0}&=6\rho_1\rho_2 V_1 V_2\frac{1}{(\rho_2 V_2+\rho_1 V_1)^2}\cdot\left[\left(\frac{V_2^2}{V_1^2}\right)^2-1\right]\\&+\frac{4\left[\rho_1^2 V_2^2-\rho_2^2 V_2^2+\rho_1\rho_2 V_1 V_2\left(\dfrac{V_2^2}{V_1^2}-1\right)\right]\cdot\left[\left(\rho_2^2 V_2^2+\rho_1^2 V_2^2\right)-\rho_1 V_1\rho_2 V_2\left(1+\dfrac{V_2^2}{V_1^2}\right)\right]}{(\rho_2 V_2+\rho_1 V_1)^8}\\&+8\frac{1}{(\rho_2 V_2+\rho_1 V_1)^4}\left[\rho_1^2 V_2^2-\rho_2^2 V_2^2+\rho_1\rho_2 V_1 V_2\left(\frac{V_2^2}{V_1^2}-1\right)\right]\cdot[\left(\rho_2^2 V_2^2+\rho_1^2 V_2^2\right)\\&+\rho_1 V_1\rho_2 V_2\left(1+\frac{V_2^2}{V_1^2}\right)]\end{aligned} \tag{10}$$

We denote the density and velocity ratios of the bottom medium to the upper medium as $\alpha_\rho=\frac{\rho_2}{\rho_1}$ and $\alpha_V=\frac{V_2}{V_1}$, respectively. Then, the two-term approximation to the reflection coefficient is obtained from Eqs. (6)–(9) as

$$\begin{aligned}R(\theta)&\approx R_0+\left(\frac{\partial R}{\partial\sin\theta}_{\sin\theta=0}\right)\sin\theta+\frac{1}{2!}\left(\frac{\partial^2 R}{\partial\sin^2\theta}_{\sin\theta=0}\right)\sin^2\theta\\&=\frac{\alpha_\rho\alpha_V-1}{\alpha_\rho\alpha_V+1}+\frac{\alpha_\rho\alpha_V(\alpha_V^2-1)}{(\alpha_\rho\alpha_V+1)^2}\sin^2\theta\\&=A+B\sin^2\theta\end{aligned} \tag{11}$$