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A numerical study of temporal shallow mixing layers using BGK-based schemes

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ABSTRACT

A numerical study of the temporal shallow mixing layers is performed. The depth-averaged shallow water equations are solved by the finite volume method based on the Bhatnagar–Gross–Krook (BGK) equation. The filtering operation is applied to the governing equations and the well-known Smagorinsky model for the subgrid-scale (SGS) stress is employed in order to present a large eddy simulation (LES). The roll-up and pairing processes are clearly shown and the corresponding kinetic energy spectra are calculated. The effects of the Froude number and the bottom friction are numerically investigated. It is shown that the growth rate of the mixing layer decreases as the Froude number increases, which is very similar to the compressible mixing layers when considering the effects of the Mach number. The numerical results also indicate that the increase in bottom friction can enhance the stability of the flows, which is physically reasonable and consistent with the theoretical and experimental findings.

1. Introduction

Mixing layer flows can be encountered in aerodynamic, atmospheric, oceanic and hydraulic engineering, where the transverse gradient in the stream-wise velocity makes the flows unstable. The study and understanding of such flows are both theoretically and practically important. According to the Fjortofts theorem [1], instability of the flow is reached in the case of an inflection point in the transverse profile of the stream-wise velocity. Kelvin–Helmholtz instabilities can therefore develop leading to horizontal vortical structures. The wavenumber of the most unstable mode and the growth rate could be predicted to some extent by linear stability analysis; see for example [2,3]. During the past decades, many experimental investigations (for example [4–7]) and numerical studies (such as [8–10]) of mixing layer flows have been carried out, which give more insights into the flows.

Open-channel flows are the turbulent wall flows with a free surface extending over the full water depth; see for example [11] for a review of such flows. A shallow mixing layer can be characterized as a combination of a plane mixing layer flow and an open-channel flow. The flow domain is bounded by a bottom and a free surface and the width of the mixing region is large compared with the water depth. Physically, the bottom of shallow flows gives two-fold effects, one is the drag force which tends to damp the flows, and the other is the small-scale turbulence generated near the bottom. Both of them will affect the horizontal coherent structures. In [12], the spatial shallow mixing layer flow was studied in detail by experiments, analytical modeling and numerical simulation.

A two-dimensional temporal mixing layer was numerically studied and analyzed in [13]. In this paper, the temporal shallow mixing layer flow is investigated by solving the depth-averaged shallow water equations. The filtering operation is applied to the governing equations and the well-known Smagorinsky model [14] for the subgrid-scale stress is employed

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in order to present a LES. The numerical method used is the BGK-based finite volume method [15,16], which is an explicit scheme with the second-order accuracy in both time and space.

The rest of the paper is organized as follows. In Section 2, the numerical methodology and the set-up of the problem are described. The numerical results are discussed in Section 3, and the concluding remarks are given in Section 4.

2. Numerical methodology and problem set-up

The filtered shallow water equations can be written as [17,18]

$$\frac{\partial \bar{h}}{\partial t} + \frac{\partial (\bar{h}\bar{\hat{u}}_i)}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial (\bar{h}\bar{\hat{u}}_i)}{\partial t} + \frac{\partial (\bar{h}\bar{\hat{u}}_i\bar{\hat{u}}_j + \delta_{ij} g\bar{h}^2/2)}{\partial x_j} = -g\bar{h}S_i^b + \frac{\partial}{\partial x_j}[\bar{h}(\nu 2\bar{\hat{S}}_{ij} - T_{ij})] - \frac{\bar{\tau}_i^b}{\rho}, \tag{2}$$

where the hat denotes the depth-averaging operator and the bar represents the filtering operator, $i = 1, 2$ and $j = 1, 2$ with 1 indicating the streaming direction and 2 indicating the cross-stream direction. h is the water depth and u_i is the velocity in x_i direction. g is the gravitational acceleration, S_i^b is the slope of the flow bed along x_i direction. ν is the kinematic viscosity of the fluid, $\delta_{ij} = 1$ when $i = j$ and 0 otherwise. The resolved strain rate tensor $\bar{\hat{S}}_{ij}$ is defined as $\bar{\hat{S}}_{ij} = (\partial\bar{\hat{u}}_i/\partial x_j + \partial\bar{\hat{u}}_j/\partial x_i)/2$. In Eq. (2), ρ is the fluid density, and $\bar{\tau}_i^b$ is the shear stress at the bed of the flow along x_i direction, which can be modeled by the quadratic friction law [14]

$$\bar{\tau}_i^b = \rho c_f \bar{\hat{u}}_i \sqrt{\bar{\hat{u}}_j \bar{\hat{u}}_j}, \tag{3}$$

where c_f is the bed friction coefficient. The subgrid-scale tensor T_{ij} represents stresses acting on the vertical plane over the entire depth due to the combined effects of filtering and depth integration, which can be expressed as

$$T_{ij} = \overline{\widehat{u_i u_j}} - \bar{\hat{u}}_i \bar{\hat{u}}_j. \tag{4}$$

A turbulence model is needed for T_{ij} to close the governing equations. Among various SGS models [19], the simplest and most widely used eddy-viscosity model is proposed by Smagorinsky in [20], where the eddy viscosity ν_t is defined by

$$\nu_t = (C_s \Delta)^2 \left(2\bar{\hat{S}}_{ij}\bar{\hat{S}}_{ij}\right)^{1/2}. \tag{5}$$

This model is employed in our numerical simulation, the Smagorinsky constant C_s is taken as $C_s = 0.065$ [21] and the filter width $\Delta = \sqrt{\Delta x_1 \Delta x_2}$ is adopted.

The filtered shallow water equations are solved by a finite volume method based on the extended BGK equation [15,22]. In this method, the fluxes for the mass and momentum across the surface of the control volume are evaluated from the solution of the BGK equation. The scheme is explicit and second-order in both time and space. It is well known that the Navier–Stokes equations can be obtained from the BGK equation in conjunction with the Chapman–Enskog expansion for low Knudsen number. The direct connection between the unfiltered shallow water equations and the extended BGK model has been established in [15], where the viscous terms in the shallow water equations are recovered from the collision term in the BGK model by setting $\nu = \sigma gh/2$ with σ the collision time. For the filtered shallow water equations, if the eddy-viscosity turbulence model is used for the SGS stress, then the filtered Eqs. (1) and (2) are mathematically equivalent to the classical unfiltered shallow water equations by replacing h, u_i and ν by $\bar{h}$, $\bar{\hat{u}}_i$ and $\nu+\nu_t$, respectively. Thus the BGK-based finite volume method can apply to solve the filtered shallow water equations directly [17]. Detailed derivation and an in-depth analysis of the BGK model for shallow water flows can be found in [15,23].

It should be noted that the BGK-based schemes have been developed and applied to a wide range of flow problems besides the free surface flows, such as the compressible flows [24], near incompressible flows [25], rarefied gas [26] and microscale gas [27] flows. One of the distinguished features for the BGK-based method is that it does not require the operator splitting of the advection and diffusion (both molecular and turbulent) terms, which may be problematic in some circumstances, see for example [28,29]. An interested reader may refer to [30] for a general review of the BGK-based schemes.

The set-up of the problem is given as follows. The gravitational acceleration is taken as $g = 9.8$ ms^{-2} and the slopes of the flow bed are assumed to be zero, i.e. $S_1^b = S_2^b = 0$. The initial mean velocity is given by

$$u_1 = U \tanh\left(\frac{2x_2}{\delta_i}\right) \text{ms}^{-1}, \qquad u_2 = 0, \tag{6}$$

which yields $u_1 = U$ ms^{-1} for $x_2 = +\infty$ and $u_1 = -U$ ms^{-1} for $x_2 = -\infty$. The vorticity thickness δ at any time is defined by

$$\delta(t) = 2U / \left[\frac{\partial \tilde{u}_1(t, x_2)}{\partial x_2}\right]_{\max}, \tag{7}$$