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Simultaneous inversion for velocity and attenuation by waveform tomography

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ABSTRACT

Seismic waveform tomography can invert for the velocity and attenuation (Q^{-1}) variations simultaneously. For this simultaneous inversion, we propose two strategies for waveform tomography. First, we analyze the contributions of the real part and the imaginary part of the gradients, associated with the velocity and attenuation parameters respectively, and determine that the combination of the real part of the gradient subvector for the velocity parameter and the imaginary part of the gradient subvector for the attenuation parameter would produce an optimal inversion result. Second, we attempt to balance the sensitivities of the objective function to the velocity and the attenuation parameters. Considering the magnitude differences between these two-type parameters in the simultaneous inversion, we apply preliminarily a normalization to both the velocity model and the attenuation model. However, for balancing their sensitivities, we further adjust the corresponding model updates using a tuning factor. We determine this tuning parameter adaptively, based on the sensitivities of these two parameters, at each iteration. Numerical tests demonstrate the feasibility and reliability of these two strategies in full waveform inversion.

1. Introduction

Seismic waveform tomography is an efficient way to obtain subsurface models, defined by various physical parameters including velocity, density and attenuation, etc. Among these parameters, velocity is the most commonly inverted parameter by waveform tomography (Gauthier et al., 1986; Brossier et al., 2009; Rao et al., 2006; Sourbier et al., 2009; Wang and Rao, 2006, 2009; Kim et al., 2011; Rao et al., 2016). However, other parameters are also important, such as the attenuation parameter Q^{-1}, which can be used as a lithology and fluid indicator. Therefore, simultaneous inversion for both the velocity and attenuation parameters is necessary for reservoir geophysics.

Waveform tomography for the attenuation Q^{-1}, or Q directly, can be implemented either in the time or frequency domain. Hicks and Pratt (2001) extracted Q parameter using frequency-domain waveform tomography. Wang (2008, Chapter 12) and Rao and Wang (2009) proposed a strategy to invert for the velocity and the attenuation Q^{-1} sequentially in waveform tomography, based on their different sensitivities. Quan and Harris (1997) and Cavalca and Fletcher (2008) derived the attenuation models using ray based tomography. Cheng et al. (2015) estimated both velocity and Q model through viscoacoustic waveform inversion in the time domain. When using waveform tomography to invert for attenuation, there is a variety of parameterizations, such as the imaginary part of the complex-valued slowness or velocity (Pratt et al., 2004; Wang, 2008; Rao and Wang, 2009; Kamei and Pratt, 2013; Rao and Wang, 2015), the inverse square-root of the complex-valued velocity (Hak and Mulder, 2011), and Q^{-1} or Q directly (Malinowski et al., 2011). In this study, we invert for Q^{-1} in the tomographic inversion.

We implement seismic waveform tomography for the velocity and attenuation parameters in the frequency domain. In frequency-domain waveform tomography, only several distinct frequencies are inverted (Wang and Rao, 2009), and the wave equation can be easily modified to include the attenuation by using a complex-valued velocity. However, when inverting for the velocity and the Q^{-1} models simultaneously, there are two fundamental questions we should pay attention to.

First, should the real part or the imaginary part of the gradient subvector be applied to calculate the model perturbation? Although the velocity and the Q^{-1} parameters are real valued, the gradient of the objective function with respect to these two parameters are complex-valued. In the velocity-only inversion, only the real part is used to update the velocity model (Wang and Rao, 2009). In two-parameter inversion, the real part of the gradient subvector with respect to the velocity is useful for the velocity model, just like in the case of the velocity-only inversion, but the imaginary part of the gradient subvector with respect to the attenuation parameter can give a better inverted

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Q^{-1} model. This is one of the strategies which will be discussed in this paper in detail.

Secondly, we know that the velocity and Q^{-1} parameters have different physical units and magnitudes, but how to effectively balance the differences and improve both parameters in a simultaneous inversion? To take care of the magnitude difference in the inversion, it is necessary to have a normalization. But a more critical issue is the difference of sensitivity to these parameters. In this paper, we propose a tuning parameter to balance the updates in the velocity model and the Q^{-1} model, and show how to properly choose such a tuning parameter in an iterative inversion.

The paper is arranged in the following sections. Section 2 summarizes briefly the frequency-domain waveform inversion theory and provides explicit expressions of gradient subvectors we concerned. Section 3 analyzes the sensitivities of the objective function with respect to the velocity and attenuation parameters. Section 4 focuses on upscaling the updates for the Q^{-1} model. Finally, Section 5 presents the numerical experiments and a discussion on the results.

2. Frequency-domain waveform tomography

The 2D viscoacoustic wave equation in the frequency domain can be expressed as (Malinowski et al., 2011)

$$\left(\frac{\omega^2}{\tilde{K}(x,z)}+\frac{\partial}{\partial x}\left(\frac{1}{\rho(x,z)}\frac{\partial}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\rho(x,z)}\frac{\partial}{\partial z}\right)\right)P(x,z,\omega)=F(x,z,\omega), \quad (1)$$

where $P(x,z,\omega)$ is the frequency domain seismic wavefield, $\rho(x,z)$ is density, $F(x,z,\omega)$ is the frequency domain source, $\tilde{K}(x,z)$ is the complex bulk modulus,

$$\tilde{K}(x,z)=\rho(x,z)\tilde{v}^2(x,z), \quad (2)$$

$\tilde{v}(x,z)$ is a complex velocity, expressed as (Blanch et al., 1995; Wang, 2008)

$$\tilde{v}(x,z)=v(x,z)\left(1-\frac{i}{2Q(x,z)}\right), \quad (3)$$

and Q is the quality factor, which is assumed to be frequency independent (Wang and Guo, 2004).

In this paper, we define a plane wave as $P(x,t) = P_0(x,t)\exp(-\alpha x)\exp[i(kx-\omega t)]$, where $\alpha = \omega/(2vQ)$.

The frequency-domain wave Eq. (1) can be represented in a matrix-vector form as

$$\mathbf{AP}=\mathbf{F}, \quad (4)$$

where $\mathbf{A}$ is the complex-valued matrix which is a function of frequencies and model properties, $\rho(x,z)$ and $\tilde{v}(x,z)$ (which includes $Q^{-1}(x,z)$), $\mathbf{P}$ is the frequency domain wavefield vector and $\mathbf{F}$ is the frequency domain source vector.

For waveform tomography, the objective function is set as

$$\phi(\mathbf{m})=\frac{1}{2}\delta\mathbf{P}^{\mathrm{H}}(\mathbf{m})\delta\mathbf{P}(\mathbf{m}), \quad (5)$$

where $\delta\mathbf{P}(\mathbf{m})\equiv\mathbf{P}_{obs}-\mathbf{P}_{cal}(\mathbf{m})$ is the difference between the observed data ($\mathbf{P}_{obs}$) and the calculated data ($\mathbf{P}_{cal}$), and H stands for the Hermitian transpose, i.e. $\delta\mathbf{P}^{\mathrm{H}}(\mathbf{m})$ is the transpose of the complex conjugate of vector $\delta\mathbf{P}(\mathbf{m})$.

The gradient, which is the first-order derivative of the misfit function with respect to model parameters, is

$$\nabla_{\mathrm{m}}\phi=-\mathbf{J}^{\mathrm{H}}\delta\mathbf{P}, \quad (6)$$

where $\mathbf{J}$ is the Fréchet derivative, and can be expressed as

$$\mathbf{J}=\frac{\partial\mathbf{P}(\mathbf{m})}{\partial\mathbf{m}}. \quad (7)$$

This Fréchet matrix can be worked out as the following (Pratt et al., 1998; Ravaut et al., 2004). Taking the first-order derivative to Eq. (4), with respect to a parameter m_k,

$$\mathbf{A}\frac{\partial\mathbf{P}}{\partial m_k}+\frac{\partial\mathbf{A}}{\partial m_k}\mathbf{P}=0, \quad (8)$$

we obtain

$$\frac{\partial\mathbf{P}}{\partial m_k}=-\mathbf{A}^{-1}\frac{\partial\mathbf{A}}{\partial m_k}\mathbf{P}. \quad (9)$$

Then the element of the gradient can be expressed as

$$\frac{\partial\phi}{\partial m_k}=\mathbf{P}^{\mathrm{H}}\left(\frac{\partial\mathbf{A}}{\partial m_k}\right)^{\mathrm{H}}\left(\mathbf{A}^{-1}\right)^{\mathrm{H}}\delta\mathbf{P}. \quad (10)$$

Assuming the subsurface model is discretized into N grids, so the matrix $\mathbf{A}$ has a dimension of $N\times N$. Meanwhile, $\partial\mathbf{A}/\partial m_k$ is also an $N\times N$ matrix. However, any of this $N\times N$ matrix $\partial\mathbf{A}/\partial m_k$ has only a single non-zero element at a grid point $k\equiv(i_x,i_z)$:

$$\left[\frac{\partial\mathbf{A}}{\partial v}\right]_{k,k}\approx\frac{-2\omega^2}{\rho_{i_x,i_z}v_{i_x,i_z}^3}, \quad (11)$$

$$\left[\frac{\partial\mathbf{A}}{\partial Q^{-1}}\right]_{k,k}\approx\frac{i\omega^2}{\rho_{i_x,i_z}v_{i_x,i_z}^2}. \quad (12)$$

The approximations above are made based on the assumption that $Q>>1$.

The gradients of the misfit function, with respect to velocity and Q^{-1} parameters, can be expressed as:

$$\begin{aligned}\frac{\partial\phi}{\partial v_k}&=\left[\frac{\partial\mathbf{A}}{\partial v}\right]_{k,k}^{*}p_k^{*}b_k,\\ \frac{\partial\phi}{\partial Q_k^{-1}}&=\left[\frac{\partial\mathbf{A}}{\partial Q^{-1}}\right]_{k,k}^{*}p_k^{*}b_k,\end{aligned} \quad (13)$$

where * represents complex conjugate, and $\mathbf{b}=(\mathbf{A}^{-1})^{\mathrm{H}}\delta\mathbf{P}$ is the back-propagated wavefield.

When $p_k^*b_k$ represents a correlation of two wavefields, then $[\partial\mathbf{A}/\partial v]_{k,k}^*$ and $[\partial\mathbf{A}/\partial Q^{-1}]_{k,k}^*$ can be considered as two weighting factors. Note that, after approximation, $[\partial\mathbf{A}/\partial v]_{k,k}^*$ has a pure non-zero real part, and $[\partial\mathbf{A}/\partial Q^{-1}]_{k,k}^*$ has a pure non-zero imaginary part.

In this paper, we will show that only the real part of the gradient subvector for the velocity parameter plays a key role in the velocity model update, and only the imaginary part of the gradient subvector for the Q^{-1} parameter plays an important role in the Q^{-1} model update:

$$\begin{aligned}\Delta v_k&\propto\mathrm{Re}\left\{-\left[\frac{\partial\mathbf{A}}{\partial v}\right]_{k,k}^{*}p_k^{*}b_k\right\},\\ \Delta Q_k^{-1}&\propto\mathrm{Im}\left\{-\left[\frac{\partial\mathbf{A}}{\partial Q^{-1}}\right]_{k,k}^{*}p_k^{*}b_k\right\}.\end{aligned} \quad (14)$$

3. Sensitivity analysis

To see the sensitivity difference between the velocity and the Q^{-1} parameter, let us first examine the variation of the misfit function with respect to the variation of these two parameters.