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General rectangular grid based time-space domain high-order finite-difference methods for modeling scalar wave propagation

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ABSTRACT

We develop the general rectangular grid discretization based time-space domain high-order staggered-grid finite-difference (SGFD) methods for modeling three-dimension (3D) scalar wave propagation. The proposed two high-order SGFD schemes can achieve the arbitrary even-order accuracy in space, and the fourth- and sixth-order accuracies in time, respectively. We derive the analytical expression of the high-order FD coefficients based on a general rectangular grid discretization with different grid spacing in all axial directions. The general rectangular grid discretization makes our time-space domain SGFD schemes more flexible than the existing ones developed on the cubic grid with the same grid spacing in the axial directions. Theoretical analysis indicates that our time-space domain SGFD schemes have a better stability and a higher accuracy than the traditional temporal second-order SGFD scheme. Our time-space domain SGFD schemes allow larger time steps than the traditional SGFD scheme for attaining a similar accuracy, and thus are more efficient. Numerical example further confirms the superior accuracy, stability and efficiency of our time-space domain SGFD schemes.

1. Introduction

Currently, the finite-difference (FD) method has been widely used in seismic forward modeling (e.g., Song and Fomel, 2011; Song et al., 2013; Tan and Huang, 2014a), seismic imaging (e.g., Dai and Schuster, 2012; Tan and Huang, 2014b) and seismic inversion (e.g., Shipp and Singh, 2002; Virieux and Operto, 2009). The early FD methods (e.g., Kelly et al., 1976; Virieux, 1984, 1986) adopted the low-order FD operator to discretize the spatial and temporal derivatives respectively, and only achieved the second-order accuracy in both time and space. The low-order FD scheme makes the simulation result suffer from great numerical dispersion. Levander (1988) kept the temporal second-order FD discretization and improved the spatial accuracy to fourth-order by using a longer stencil length. The spatial fourth-order FD scheme achieves a better tradeoff between accuracy and efficiency, and thus becomes a favored tool for modeling seismic wave propagation (e.g., Robertsson et al., 1994; Graves, 1996; Moczo et al., 2000). The spatial accuracy is further improved by using higher-order FD operators (e.g., Holberg, 1987; Fornberg, 1987; Chu and Stoffa, 2012; Dong et al., 2013).

Although the arbitrary even-order accuracy in space has been achieved by increasing the spatial FD stencil length, the temporal second-order discretization is still popular, because of its relatively low requirements of memory storage. For the standard temporal second-order FD discretization scheme, one usually has to adopt a small time step to suppress temporal dispersion during long distance wave propagation. To improve the temporal approximation accuracy without significantly increasing the memory storage requirement, Dablain (1986) developed a temporal fourth-order acoustic FD modeling scheme by applying the Lax–Wendroff approach (Lax and Wendroff, 1964). Crase (1990) adopted a similar approach to develop an elastic FD modeling scheme which is accurate to arbitrary order in both space and time. Blanch and Robertsson (1997) proposed the Lax–Wendroff approach based FD method for modeling anelastic wave propagation. Chen (2011) discussed the stability of the Lax–Wendroff approach based temporal fourth-order FD method. The Lax–Wendroff approach based FD methods achieve the high-order temporal accuracy by replacing the high-order temporal derivatives with the spatial derivatives using the wave equation. However, the FD methods involve expensive calculations of high-order spatial derivatives. Moreover, the mentioned FD methods are only up to the tenth-order accuracy in space.

The newly developed time-space domain FD methods can achieve both temporal and spatial high-order accuracies by determining the FD coefficients from the dispersion relation (e.g., Liu and Sen, 2009; Liu et al., 2014; Fang et al., 2014). Depending on the adopted approach for calculating the FD coefficients, the time-space domain FD methods can be further classified into two categories, the Taylor-series expansion

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(TE) based methods (e.g., Tan and Huang, 2014a; Liu and Sen, 2009) and optimization based methods (e.g., Tan and Huang, 2014c; Wang et al., 2014; Chen et al., 2015; Chen et al., 2016). The optimized FD methods can attain a better accuracy than the TE based methods, but require extra computational efforts for optimizing the FD coefficients. The coefficients of the time-space domain FD methods are velocity dependent, which results in separated optimization for each distinct velocity in heterogeneous media. Contrarily, the coefficients of the TE based FD methods are expressed analytically. The computational cost for determining the TE based coefficients is smaller than that for determining the optimized coefficients. Liu and Sen (2013) developed a TE based time-space domain FD method with same arbitrary even-order accuracy in space and time by adopting new FD stencils, however, they only discussed the 2D modeling problem, and did not derive the analytical expressions of the FD coefficients. Tan and Huang (2014a) proposed the TE based spatial $2M$-th-order, temporal 4th- and 6th-order SGFD methods, denoted by $(2M,4)$ and $(2M,6)$, respectively. They derived the analytic expressions of the FD coefficients in both 2D and 3D.

Although various time-space domain FD methods have been proposed to improve computational efficiency of seismic wave modeling, most of them are based on a square or cubic grid discretization with equal grid spacing in all axial directions (e.g., Tan and Huang, 2014a; Wang et al., 2014). However, for modeling seismic wave propagation in some special geological bodies, e.g., the models containing thin layers but much longer in width, a general rectangular gird discretization with unequal horizontal and vertical grid spacing is particularly useful for reducing memory requirement and computational cost. Moreover, for modeling some boundary conditions, e.g., the free surface boundary condition, a denser sampling in the vertical direction than the horizontal direction is necessary (e.g., Kosloff et al., 1990; Mittet, 2002). Inspired by this motivation, we assume in this paper a general rectangular grid discretization with unequal grid spacing in each axial direction, and develop the $(2M,2N,2L,4)$ and $(2M,2N,2L,6)$ SGFD schemes to simulate 3D scalar wave equation, where $2M$, $2N$, and $2L$ represent the stencil lengths in the x-, z-, and y-axial directions respectively in the 3D Cartesian coordinate space. Accordingly, the traditional SGFD scheme with the temporal second-order accuracy is denoted by $(2M,2N,2L,2)$.

The paper is organized as follows. We first introduce the temporal fourth-, sixth-order, and spatial arbitrary even-order SGFD discretization of the first-order spatial derivatives on a general rectangular grid. Then, we derive the analytical expressions of the FD coefficients using the TE approach. Next, we discuss the stability conditions, conduct an accuracy analysis, and theoretically compare the computational cost of our $(2M,2N,2L,4)$ and $(2M,2N,2L,6)$ SGFD schemes with the traditional $(2M,2N,2L,2)$ scheme. This is followed by numerical examples. Finally we draw some conclusions.

2. Material and methods

The velocity-pressure formulation of scalar wave equation is given by

$$\begin{cases} \dfrac{\partial p}{\partial t} + K\nabla \cdot \boldsymbol{v} = 0, \\ \dfrac{\partial \boldsymbol{v}}{\partial t} + \dfrac{1}{\rho}\nabla p = 0, \end{cases} \tag{1}$$

where $\rho(\boldsymbol{x})$ is the density, $K(\boldsymbol{x}) = \rho(\boldsymbol{x})v^2(\boldsymbol{x})$ is the bulk modulus, $v(\boldsymbol{x})$ represents velocity, $p(\boldsymbol{x},t)$ represents the pressure, $\boldsymbol{v} = [v_x(\boldsymbol{x},t), v_y(\boldsymbol{x},t), v_z(\boldsymbol{x},t)]^{\mathrm{T}}$ denotes the particle velocity vector, $\boldsymbol{x} = (x,y,z)$ represents 3D Cartesian coordinate system, and t denotes time.

2.1. Time-space domain discretization of first-order spatial derivatives

2.1.1. The (2M, 2N, 2L, 4) discretization scheme

The temporal fourth-order and spatial sixth-order SGFD stencil for discretizing $\partial/\partial x$ is depicted in Fig. 1, where 8 off-axial grid points are included in the stencil. If one wants to achieve a higher-order spatial accuracy, one needs to include more grid points in the difference direction. Generally, the temporal fourth-order and spatial arbitrary even-order discretization of the first-order derivatives at the position of $(0,0,0)$ leads to (Tan and Huang, 2014a)

$$\frac{\partial p_{0,0,0}}{\partial x} \approx \frac{1}{h_x}\left\{ \begin{array}{l} \sum\limits_{m=1}^{M} c^{x}_{m,0,0}\left(p_{m-\frac{1}{2},0,0} - p_{-m+\frac{1}{2},0,0}\right) + c^{xy}_{1,1,0}\left(p_{\frac{1}{2},1,0} - p_{-\frac{1}{2},1,0} + p_{\frac{1}{2},-1,0} - p_{-\frac{1}{2},-1,0}\right) \\ + c^{xz}_{1,0,1}\left(p_{\frac{1}{2},0,1} - p_{-\frac{1}{2},0,1} + p_{\frac{1}{2},0,-1} - p_{-\frac{1}{2},0,-1}\right) \end{array} \right\}, \tag{2}$$

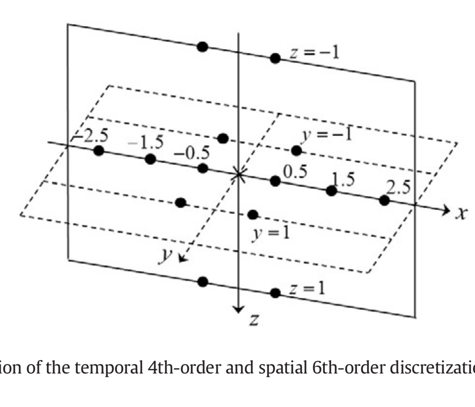

Fig. 1. Illustration of the temporal 4th-order and spatial 6th-order discretization of $\partial/\partial x$ in 3D.