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Approximate iterative operator method for potential-field downward continuation

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ABSTRACT

An approximate iterative operator method in wavenumber domain was proposed to improve the stability and accuracy of downward continuation of potential fields measured from the ground surface, marine or airborne. Firstly, the generalized iterative formula of downward continuation is derived in wavenumber domain; then, the transformational relationship between horizontal second-order partial derivatives and continuation is derived based on the Taylor series and Laplace equation, to obtain an approximate operator. By introducing this operator to the generalized iterative formula, a rapid algorithm is developed for downward continuation. The filtering and convergence characteristics of this method are analyzed for the purpose of estimating the optimal interval of number of iterations. We demonstrate the proposed method on synthetic data, and the results validate the flexibility of the proposed method. At last, we apply the proposed method to real data, and the results show the proposed method can enhance gravity anomalies generated by concealed orebodies. And in the contour obtained by making our proposed method results continue upward to measured level, the numerical results have approximate distribution and amplitude with original anomalies.

1. Introduction

Downward continuation (DWC) is a useful approach to enhance potential field and their sources. Its computational accuracy and speed directly influence the later data processing (Oruç and Keskinsezer, 2008; Zhou, 2015). Therefore, investigating a novel DWC method which stability, accuracy and speed are higher has important significance to explore minerals, oil gas and geological structure. DWC is a typical ill-posed problem and unstable. In other words, it is influenced by the high frequency effects arising from noise (Dean, 1958; Ning et al., 2005). This ill-posed problem is often solved in two computational domains in mathematical and geophysical fields.

Methods in spatial domain are accurate and widely applied to continue the potential field data downward, special inversion of the satellite data (Xu, 1992; Xu et al., 2006; Eshagh and Sjöberg, 2011; Eshagh, 2011; Eshagh et al., 2013). In general, these methods can be categorized into two groups: (a) direct methods (Tikhonov, 1963; Scherzer, 1993; Bouman, 1998) and (b) iterative methods. Some common iterative methods are generally based on regularization theory, and combine matrix operations (e.g. truncated singular value decomposition and variance component estimation) and iterative scheme to get more accurate results (Björck, 1988; Xu, 1992; Hämarik and Tautenhahn, 2001; Koch and Kusche, 2002; Jensen and Hansen, 2007; Xu et al., 2006; Eshagh, 2011; Eshagh et al., 2013). However, how to estimate the optimal regularization parameters is often difficult (Kilmer and O'Leary, 2001). For solving this problem, scholars have proposed some methods like minimizing the estimated mean square error (Xu, 1998), L-curve (Calvetti et al., 2000; Zeng et al., 2013) and generalized cross validation (Wahba, 1976; Hansen, 1998; Xu, 2009) etc. However, there is a great amount of calculation against rapid transformation in the process of matrix operations and estimating the optimal regularization parameter in spatial domain.

The DWC methods in wavenumber domain are accurate and rapid without matrix operations. In general, these methods can be categorized into direct methods and iterative methods as well. Direct methods mainly include ideal window filters (Ku et al., 1971), Wiener filters (Pawlowski, 1995; Trompat et al., 2003) and regularization filters (Li et al., 2013; Zeng et al., 2014). Among them, most common regularization filters need Tikhonov regularization operator in series into DWC operator, and estimate the optimal regularization parameter by L-curve as well (Abedi et al., 2013; Li et al., 2013). The iterative method in wavenumber domain was proposed for DWC by Strakhov and Devitsyn (1965). Since the 1970s, using iterative concepts to improve DWC of potential field data measured from the ground surface, marine or airborne has become the main research direction of Chinese scholars. Yang et al. (1999) proposed the apparent depth filtering method (ADFM), combined the finite difference method, Taylor series and iterative smooth

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filtering. Liu et al. (2009) presented the integral iterative method (IIM) for DWC. Wang et al. (2011) proposed the Taylor series iterative method (TSIM), in which Taylor series was performed on the theoretical operator of DWC. These iterative methods have their respective advantages in the computational stability, accuracy and speed. However, they do not present any method to estimate optimal iterations, and the effect of iterative methods largely depends on the speed and accuracy of estimating number of iterations (Yao et al., 2012).

In this study, on the basis of previous research, an approximate iterative operator method (AIOM) for DWC of potential field data is proposed by introducing a derived approximate operator into the iterative algorithm in wavenumber domain. After analyzing the filtering and convergence properties of this method, the correlation coefficient curve is discussed to determine the optimal interval of number of iterations. This method can effectively enhance local anomalies and provide higher-accuracy data for other later related processing rapidly.

2. Theory

Potential-field DWC can be written in wavenumber domain as follow:

$$F_B = \tilde{F}_A \cdot \psi_{down} \tag{1}$$

where $\tilde{F}_B$ represents the Fourier transform spectrum of the downward continued anomaly; $\tilde{F}_A$ represents the Fourier transform spectrum of original anomaly at the measured level; and $\psi_{down}$is the theoretical operator of DWC.

In order to suppress the amplification effect of the theoretical operator ψ_{down} in Eq. (1) at mid-high frequency of $\tilde{F}_A$ and improve the computational stability, we could design a new approximate operator φ_{down}which always has the same filtering properties as theoretical operator ψ_{down} at low frequencies, and has appropriate suppressions at mid-high frequencies. Apparently, the φ_{down}can not only enhance the stability of DWC inevitably, but also suppresses some useful mid-high frequencies of $\tilde{F}_A$. Thus it is required to adopt the following iterative algorithm to get more frequencies for the results.

Approximate operator φ_{down} is used for DWC of the potential field data at the measured level A to obtain the first-order approximate spectrum at target level B, as follow:

$$\tilde{F}_B^{(1)} = \tilde{F}_A \varphi_{down} \tag{2}$$

Eq. (2) is then multiplied by upward continuation (UWC) operator ϕ_{up} on both sides simultaneously, to obtain the first-order approximate spectrum of the UWC of $\tilde{F}_B^{(1)}$ at the measured level:

$$\tilde{F}_A^{(1)} = \tilde{F}_B^{(1)} \phi_{up} \tag{3}$$

Meanwhile, the first-order residual spectrum at the measured level can be shown as follow:

$$\delta\tilde{F}_A^{(1)} = \tilde{F}_A - \tilde{F}_A^{(1)} \tag{4}$$

Moreover, the approximate operator φ_{down} is used to continue the first-order residual spectrum $\delta\tilde{F}_A^{(1)}$ downward to target level B, to determine the first-order residual spectrum:

$$\delta\tilde{F}_B^{(1)} = \delta\tilde{F}_A^{(1)} \varphi_{down} \tag{5}$$

Eq. (5) is then used to correct the first-order approximate spectrum ($\tilde{F}_B^{(1)}$), and determine the second-order approximate spectrum at the level B:

$$\tilde{F}_B^{(2)} = \tilde{F}_B^{(1)} + \delta\tilde{F}_B^{(1)} \tag{6}$$

Above steps are repeated n times to determine the following iterative equation:

$$\begin{cases} \tilde{F}_B^{(n)} = \varphi_{down}\tilde{F}_A + \left(1-\varphi_{down}\phi_{up}\right) \\ \qquad \tilde{F}_B^{(n\text{-}1)}\tilde{F}_B^{(1)} = \varphi_{down}\tilde{F}_A \end{cases} \tag{7}$$

where $\tilde{F}_B^{(n)}$ and $\tilde{F}_B^{(n-1)}$ represent the spectrums of downward continued anomalies with n and n - 1 iterations. Mathematical induction (Movshovitz-Hadar, 1993) is performed on Eq. (7), and it is easy to determine the following generalized formula of the iterative method for DWC in wavenumber domain:

$$\tilde{F}_B^{(n)} = \left[1-\left(1-\varphi_{down}\phi_{up}\right)^n\right] \cdot \psi_{down} \cdot \tilde{F}_A \tag{8}$$

If we let g_A represent the potential field data at measured level A, and $g_A^{(n)}$ represent the inverse Fourier transformation of approximate spectrum $\tilde{F}_A^{(n)}$ which is obtained by continuing $\tilde{F}_B^{(n)}$ upward to measured level A. Until the condition $\max|g_A - g_A^{(n)}| < \varepsilon$ is satisfied (ε is the given permission error), the iteration is terminated. Finally, inverse Fourier transformation is performed on $\tilde{F}_B^{(n)}$ to obtain downward continued anomaly at target level B. However, due to the fact that ε is a given value, the stop condition of iterative method is difficult to grasp in practice, a feasible scheme of estimating the optimal interval of number of iterations will be proposed in following sections.