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BTTB–RRCG method for downward continuation of potential field data

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ABSTRACT

This paper presents a conjugate gradient (CG) method for accurate and robust downward continuation of potential field data. Utilizing the Block-Toeplitz Toeplitz-Block (BTTB) structure, the storage requirement and the computational complexity can be significantly reduced. Unlike the wavenumber domain regularization methods based on fast Fourier transform, the BTTB-based conjugate gradient method induces little artifacts near the boundary. The application of a re-weighted regularization in a space domain significantly improves the stability of the CG scheme for noisy data. The synthetic data with different levels of added noise and real field data are used to validate the effectiveness of the proposed scheme, and the computed results are compared with those based on recently proposed wavenumber domain methods and the Taylor series method. The simulation results verify that the proposed scheme is superior to the existing methods considered in this study in terms of accuracy and robustness. The proposed scheme is a powerful computational tool capable of applications for large scale data with modest computational cost.

1. Introduction

Downward continuation is frequently applied to enhance the potential field data. It provides geological information at low elevation by using the field data from high elevation. In recent years, as the aerogravity and magnetic surveys become more widely used in prospecting (Zhang et al., 2015), it is desirable to develop efficient and robust downward continuation methods to deal with large amounts of aeropotential field data. According to the physical law, the potential field data at higher elevation contains dim geophysical information, which makes the data less valuable. The potential field data can be enhanced by using a downward continuation technique, such that the potential field at lower elevation or even underground within the harmonic source-free region (Pašteka et al., 2012) can be effectively estimated.

In the wavenumber domain (Fourier spectral domain), the continuation can be carried out by multiplying a continuation factor with the spectrum of the observation data. Unfortunately, the downward continuation factor grows rapidly as the continuation distance increases. The high frequency components including noise in the observation data will be amplified and thus resulting a severe polluted solution. Therefore, using downward continuation in a wavenumber domain is an inherently unstable process. Using appropriate filters or constrains, stable downward continuation can be constructed. Dean (1958)) proposed a method to constrain the high frequency components. The use of a Wiener filter is investigated in Clarke (1969) and Pawlowski (1995). Recently, Pašteka et al. (2012) propose a robust wavenumber domain method where the filter is designed based on the characteristics of Tikhonov regularization, Zeng et al. (2013) use an adaptive iterative Tikhonov method to apply a Tikhonov filter in each iteration in a wavenumber domain. The advantage of a wavenumber domain method is that the downward continuation process can be accelerated by fast Fourier transform (FFT). It has been proved that an appropriate designed filter can guarantee the accuracy and stability of downward continuation even for noisy data (Pašteka et al., 2012; Zeng et al., 2013).

Another type of downward continuation method is based on the Taylor expansion, where the potential field at one elevation can be expanded by the potential field and its vertical derivative terms at another elevation. The success of the Taylor series method depends on the accuracy and stability in computing the vertical derivative terms. Fedi and Florio (2002, 2001) propose the ISVD method, where the odd vertical derivatives can be computed in a stable way, and the even order vertical derivative can be efficiently computed by finite difference. Zhang et al. (2013) propose a truncated Taylor series iterative scheme to achieve robust and stable downward continuation. Ma et al. (2013) compute the downward continuation by adding an upward continuation and a second vertical derivative at the observation plane, and the scheme can also be converted to an iterative version. The Taylor series method is capable of providing very accurate solution when the data are relatively clean, moreover the iterative Taylor series method usually has a fast convergence rate.

It should be noted that both the wavenumber domain methods and the Taylor series methods can be accelerated by FFT. However, the FFT itself inherently introduces an artifact, and the FFT-induced artifact can be seen in many existing methods, this is particularly obvious in some iterative wavenumber domain methods (Zeng et al., 2013). The FFT-induced error in the downward continuation process has already

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been studied by researchers in Trompat et al. (2003); Cooper (2004); and Pašteka et al. (2012). To resolve the difficulty, either extrapolation is needed to extend the original data (Pašteka et al., 2012), or a smaller window should be used to exclude the results near the boundary. For the Taylor series methods, besides the FFT-induced error, another problem is the robustness for the noisy data. Although the ISVD method (Fedi and Florio, 2002; Fedi and Florio, 2001) can be applied to compute the odd derivatives in a stable way, the even derivatives are still computed by the standard finite difference which is sensitive to the noise. Another iterative method such as that based on Taylor series has a similar problem (Zhang et al., 2013; Ma et al., 2013). Without a denoising procedure, it is hard to apply the Taylor series methods for the field data with relatively large noise.

In summary, the FFT provides an efficient computation with the numerical complexity of order $n \log n$, where n is the number of unknowns. To apply the FFT, a continuation process including the regularization is usually converted into the wavenumber domain. For this reason, regularized downward continuation in space domain has seldom been investigated. Zhang and Wong (2015) propose a numerical scheme for 3D gravity field inversion, where a special algebraic structure called Block-Toeplize Toeplize-Block (BTTB) matrix is utilized to make the scheme efficient.

In this paper, we consider the conjugate gradient (CG) method utilizing the BTTB structure for downward continuation problem. The BTTB structure is derived from the downward continuation formulation in space domain, and it has the same numerical efficiency as the FFT-based methods. However, compared with the FFT-based methods, the proposed method induces very small artifact near the boundary, such that neither extrapolation nor tailoring process is required to reduce the boundary error. The characteristic of the BTTB structure allows the use of an iterative scheme without accumulating the error near the boundary. Combining the BTTB structure with the re-weighted regularized conjugate gradient method (BTTB–RRCG), a stable downward continuation method can be constructed. Here, all formulations are in time domain, such that various space domain regularization stabilizers can be applied. We compare the proposed computational scheme with other recently proposed schemes for downward continuation. The simulation results for synthetic and field data demonstrate that the proposed scheme is more accurate and robust for applications using clean and noisy data.

In Section 2, the formulation of downward continuation is presented. We briefly introduce the Tikhonov regularized method (TR) (Pašteka et al., 2012), adaptive iterative Tikhonov method (AIT) (Zeng et al., 2013), and stable Taylor series methods (ITS) (Ma et al., 2013). Section 3 focuses on the proposed BTTB–RRCG scheme. In Section 4, synthetic field data are used to validate the proposed numerical scheme, and the result is compared with those obtained by the TR, AIT and ITS methods. A Gaussian noise from 0.1% to 5% of the maximum magnitude of the synthetic data is added to test the robustness. The error is analyzed by using RMS and the relative error in terms of L-2 norm and L-∞ norm. Particularly, the FFT-induced error near the boundary is investigated. In Section 5, we apply the proposed scheme to test cases with field data, and similar to the synthetic cases, the result is compared with other existing methods.

2. Mathematical background

The relationship between the potential field data at two observation planes is given by Blakely (1996):

$$\mathbf{T}(x,y,h_0) = \frac{h_0-h}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{T(x',y',h)}{\left[(x-x')^2+(y-y')^2+(h-h_0)^2\right]^{3/2}}, \quad (1)$$

where x and y are the horizontal coordinates, $T(x,y,h_0)$ is the observation field at higher elevation h_0, and $T(x,y,h)$ is the unknown field at lower elevation h, such that $h_0>h$. The downward continuation process is to seek $T(x,y,h)$ at lower elevation from the potential field $T(x,y,h_0)$ at higher elevation.

Denote the kernel as K, the integral Eq. (1) can be converted into the following convolution form

$$\mathbf{T}(x,y,h_0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{K}(x-x',y-y',h_0-h)\mathbf{T}(x',y',h)dx'dy', \quad (2)$$

which can be further simplified as

$$\mathbf{T}(h_0) = \mathbf{K} * \mathbf{T}(h), \quad (3)$$

where * denotes the convolution. According to the convolution theorem,

$$\mathcal{F}(\mathbf{T}(h_0)) = \mathcal{F}(\mathbf{K} * \mathbf{T}(h)) = \mathcal{F}(\mathbf{K})\cdot\mathcal{F}(\mathbf{T}(h)), \quad (4)$$

therefore,

$$\mathbf{T}(h_0) = \mathcal{F}^{-1}(\mathcal{F}(\mathbf{K})\cdot\mathcal{F}(\mathbf{T}(h))). \quad (5)$$

Since

$$\mathcal{F}(\mathbf{K}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{K}(x,y)e^{-2\pi i(ux+vy)}dxdy = e^{-(h_0-h)\sqrt{u^2+v^2}}, \quad (6)$$

denote $\mathbf{T}(h_0)$ and $\mathbf{T}(h)$ by $\mathbf{T}_{h_0}$ and $\mathbf{T}_h$, respectively, then Eq. (3) can be rewritten into the following matrix form

$$\boldsymbol{T}_{h_0} = \boldsymbol{F}^{-1}\boldsymbol{\Lambda}\boldsymbol{F}\boldsymbol{T}_h, \quad (7)$$

where $\mathbf{F}$ and $\mathbf{F}^{-1}$ are the Fourier matrices corresponding to a 2D Fourier transform, and Λ is the continuation kernel $\mathbf{K}$ in a wavenumber domain given by Eq. (6). Consider h_0-h>0, the kernel $e^{-(h_0-h)\sqrt{u^2+v^2}}$ is stable, since the high frequency component can be compressed. This explains why the upward continuation is a stable process.

According to Eq. (7), the most straightforward way to conduct a downward continuation is

$$\mathbf{T}_h = \mathbf{F}^{-1}\Lambda^{-1}\mathbf{F}\mathbf{T}_{h_0}, \quad (8)$$

where Λ^{-1} is given by $e^{(h_0-h)\sqrt{u^2+v^2}}$. Obviously, since h_0-h>0, the kernel given by $e^{(h_0-h)\sqrt{u^2+v^2}}$ will amplify all frequency components in $\mathbf{T}_{h_0}$, such that the solution of $\mathbf{T}_h$ will be polluted by the high frequency component or noise in $\mathbf{T}_{h_0}$. Denote $\mathbf{T}_h$ by $\mathbf{T}$, Eq. (8) can be rewritten in a simplified form as

$$\hat{\mathrm{T}} = \Lambda^{-1}\hat{\mathrm{T}}_{h0}, \quad (9)$$

where $\hat{\mathrm{T}}$ and $\hat{\mathrm{T}}_{h_0}$ are the potential field in wavenumber domain with heights h and h_0.

Therefore, a downward continuation is an inherently unstable process, and conventionally, there are mainly two approaches to resolve this issue: Tikhonov regularization in wavenumber domain and the Taylor series methods.

2.1. Wavenumber domain Tikhonov regularization method

Let us denote the downward continuation formulation (Eq. (1)) into the following form:

$$\mathbf{T}_{h_0} = \mathbf{A}\mathbf{T}, \quad (10)$$

where $\mathbf{A}$ is the upward continuation operator. As we have shown before, solving Eq. (10) is an ill-posed problem, which is equivalent to compute Eq. (8). Tikhonov and Arsenin (1977) propose an effective way to