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Estimation of subsurface geomodels by multi-objective stochastic optimization

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ABSTRACT

We present a new method to estimate subsurface geomodels using a multi-objective stochastic search technique that allows a variety of direct and indirect measurements to simultaneously constrain the earth model. Inherent uncertainties and noise in real data measurements may result in conflicting geological and geophysical datasets for a given area; a realistic earth model can then only be produced by combining the datasets in a defined optimal manner. One approach to solving this problem is by joint inversion of the various geological and/or geophysical datasets, and estimating an optimal model by optimizing a weighted linear combination of several separate objective functions which compare simulated and observed datasets. In the present work, we consider the joint inversion of multiple datasets for geomodel estimation, as a *multi-objective optimization problem* in which separate objective functions for each subset of the observed data are defined, followed by an *unweighted simultaneous* stochastic optimization to find the set of best compromise model solutions that fits the defined objectives, along the so-called "Pareto front". We demonstrate that geostatistically constrained initializations of the algorithm improves convergence speed and produces superior geomodel solutions. We apply our method to a 3D reservoir lithofacies model estimation problem which is constrained by a set of geological and geophysical data measurements and attributes, and assess the sensitivity of the resulting geomodels to changes in the parameters of the stochastic optimization algorithm and the presence of realistic seismic noise conditions.

1. Introduction

Subsurface geomodel estimation is a fundamental practice in many Earth science disciplines: hydrology and ground water analyses, geothermal studies, exploration and recovery of fossil fuel energy resources, and CO_2 geosequestration, among others. Yet the construction of physically realistic subsurface models which "match" a finite set of data measurements at the Earth's surface (or in boreholes) continues to challenge researchers and practitioners (e.g., Tarantola, 1987; Schwarzbach et al., 2005).

To model the subsurface conditions accurately in three dimensions, a variety of geological data (e.g., core measurements, well logs and geological maps) and geophysical data measurements (e.g., seismic, resistivity and gravity) is required. Even the most sophisticated algorithms may fail to produce convincing results, however, because each information source has its limitations (Friedel, 2003; Kozlovskaya et al., 2007). We believe that a realistic subsurface model can only be obtained by reconciling several types of data measurement. For instance, geological data may describe the general distribution of subsurface rock properties, but other factors — the limited lateral coverage of wells and the associated heterogeneous nature of most of the properties, for example — may lead to highly uncertain models (Saussus and Sams, 2012). In this context, geophysical data might add new information due to its high resolution spatial coverage, not only for the definition of the model framework and geometry, but also for both the discrete and continuous property modeling processes (Doyen, 2007). Optimal results are achieved if the final geomodels simultaneously agree with the multiple objectives that incorporate each of the complementary datasets. *Subsurface geomodel estimation then becomes a nonlinear optimization problem with multiple objective statements and datasets.*

Conventionally, there are two broad approaches to multi-objective optimization (Kozlovskaya et al., 2007):

1. Independent inversion of each dataset to obtain different self-directed (individual) models, which are then combined and averaged to produce the final model result.
2. Simultaneous joint inversion of multiple datasets which directly produce the final model result.

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A common approach to joint inversion of multiple datasets is to apply weighted summation of the objective functions defined for each subset of the assimilated datasets to obtain a global objective function:

$$\text{Global objective function} = \sum_i w_i Obj_i, \tag{1}$$

where Obj_i is the ith objective function statement, and w_i is the corresponding ith weighting factor. The global objective function is then optimized using one of several classical inversion techniques to find the best-fitting model (Miettinen, 1999; Moghadas et al., 2010). This approach poses several challenges: firstly, the difficulty in determining the optimal set of weights for each of the objective functions; secondly, the need for several optimization-runs to find a different solution for each set of trial weights; and finally the difficulties in identifying non-convex Pareto fronts (Hajizadeh, 2011). All of these issues may be subject to user bias.

In this study, we demonstrate that a joint inversion problem of this kind for subsurface geomodel estimation can be solved as a *simultaneous multi-objective optimization problem*. The proposed method consists of two steps. First, all relevant problem domain knowledge and prior information are assimilated in order to generate an ensemble of possible initial geomodels. Second, a multi-objective optimization problem is designed to update the initial ensemble of geomodels such that multiple objective functions are defined and tested against several sets of observed data. An ensemble-based stochastic search technique is then used to find the best compromise model solutions among all of the components of the objective function vector (found along the optimal Pareto front), in a single optimization run.

Multi-objective optimization problems are commonly found in computer science (e.g., Ferrer et al., 2012; Balaprakash et al., 2014), physics (e.g., Di Barba et al., 2014), economics (Bamufleh et al., 2013) and various engineering disciplines (e.g., Yildiz and Solanki, 2012; Asadi et al., 2012; Ahmadi et al., 2013; Dehghanian et al., 2013). Many Earth science inverse problems are nonlinear optimization problems with multiple constraining objective function statements and datasets that cannot be solved efficiently using a single-objective optimization approach. This is mainly due to the difficulty of choosing an optimal weighting scheme for each term in the weighted summation of the objective functions. Due to such limitations in traditional optimization approaches, various multi-objective optimization methods have been developed to solve complex geoscience inverse problems, especially if they do not require specification of objective function weights (e.g., Ray and Sarker, 2007; Carbone et al., 2008; Heyburn and Fox, 2010; Han et al., 2011). In this study, we use an evolutionary algorithm to address the multi-objective geomodel estimation problem. Historically, algorithms of this type have been successfully implemented in a number of geological and geophysical inversion problems (Wilson and Vasudevan, 1991; Gallagher et al., 1991; Sen and Stoffa, 1992). Combining the multi-objective concept with evolutionary algorithms leads to powerful global optimization algorithms that have attracted considerable attention from researchers in various fields due to their ability to find a set of optimal solutions (Singh et al., 2008). Some examples of the application of multi-objective evolutionary algorithms in various geoscience disciplines have been reported; for example, Moorkamp et al. (2007) implemented a particular kind of multi-objective evolutionary algorithm for the joint inversion of teleseismic and magnetotelluric datasets. We note that other bio-inspired heuristic optimization techniques have been reported in the context of multi-objective optimization problems (Lobato et al., 2014). Representative methods include particle swarm optimization (e.g., del Valle et al., 2008; Srivastava and Agarwal, 2010), artificial immune systems (Coello and Cortés, 2005), ant colony optimization (Guntsch, 2004), bee colony algorithms (Pham et al., 2006) and firefly colony algorithms (Yang, 2010).

In this paper we demonstrate the applicability of our method using the example of a 3D reservoir lithofacies modeling problem conditioned to seismic attributes (P- and S-wave impedance volumes) and prior geological knowledge. Since valuable information is contained in both P- and S-wave velocity and/or impedance datasets, we develop and test a multi-objective optimization method to find the best compromise geomodel solutions that simultaneously honor these two sets of inverted seismic attributes, in addition to the prior geological information.

This paper is structured as follows. First, we review the basic concepts of multi-objective optimization problems in Section 2.1 and evolutionary algorithms in Section 2.2. Then we illustrate the properties of multi-objective evolutionary algorithms and multi-criteria decision making in Sections 2.3 and 2.4 respectively. We introduce our new method for geomodel estimation and discuss its associated characteristics in Section 3, followed by its application to a 3D test model in Section 4. Finally, we present qualitative and quantitative analyses of the results in Section 5, and in Section 6 we summarize the conclusions of this study.

2. Theory

2.1. Multi-objective optimization

Given a set of real-valued objective functions $f_i(\mathbf{x})$ dependent on a real-valued decision variable $\mathbf{x}$, the general form of a multi-objective optimization problem can be written as:

$$\left.\begin{array}{lll} \text{Optimize} & \boldsymbol{f_i}(\boldsymbol{x}), & i = 1, \ldots, k; \\ \text{subject to} & h_p(\boldsymbol{x}) = 0, & p = 1, \ldots, P; \\ & g_m(\boldsymbol{x}) \geq 0, & m = 1, .., M; \\ & x_j^L < x_j < x_j^U, & j = 1, \ldots, n. \end{array}\right\} \tag{2}$$

These expressions state that a multi-objective optimization problem can be defined as finding a set of solution parameters (or decision variables) $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ which optimizes (minimizes or maximizes) a set of objective functions $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})]^T$ subject to P equality constraints $h_p(\mathbf{x}) = 0$, and M inequality constraints $g_m(\mathbf{x}) \geq 0$, where n is the number of solution parameters and k is the number of the objective functions. The variable bounds constrain each solution parameter x_j to take a value within a lower x_j^L bound and an upper x_j^U bound. The restrictions imposed by the constraint functions and variable bounds define the feasible region $\mathbf{F}$ within the search space $\mathbf{S}$ such that ($\mathbf{F} \subseteq \mathbf{S}$), and any $\mathbf{x} \in \mathbf{F}$ yields a feasible solution (Miettinen, 1999; Deb, 2001).

The concept of optimality in multi-objective optimization problems differs from that used in single-objective optimization problems; for the latter there is usually one global optimum solution; however, in multiple objective optimization problems, many optimal solutions are possible, depending on the relative importance (weight) of each objective. A unique optimum solution ($\mathbf{x}^*$) for a multi-objective optimization problem (Fig. 1, black circle solution) is a notional concept that is seldom obtainable in practical situations. As a result, it is necessary to be clear about what 'optimal solution' means in the case of multiple objectives: rather than searching for a single optimal solution, in practice it may be more appropriate to search for the set of optimal solutions that represents the best compromise or 'trade-off' between all of the feasible solutions (Coello et al., 2007).

In the context of multi-objective optimization problems, three important concepts need to be defined (Deb, 2001; Mohamed et al., 2012):

1) *Domination*: A solution $\mathbf{x}_1$ is said to dominate another solution $\mathbf{x}_2$ if both of the following conditions exist:
  - $\mathbf{x}_1$ is no worse than $\mathbf{x}_2$ for all objectives:

$$f_i(\boldsymbol{x}_1) \leq f_i(\boldsymbol{x}_2), \forall i = 1, 2, \ldots, k. \tag{3}$$

  - $\mathbf{x}_1$ is better than $\mathbf{x}_2$ for at least one objective:

$$f_i(\boldsymbol{x}_1) < f_i(\boldsymbol{x}_2), \exists i = 1, 2, \ldots, k. \tag{4}$$