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Gravity inversion and uncertainty assessment of basement relief via Particle Swarm Optimization

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ABSTRACT

Gravity inversion is a classical tool in applied geophysics that corresponds, both, to a linear (density unknown) or nonlinear (geometry unknown) inverse problem depending on the model parameters. Inversion of basement relief of sedimentary basins is an important application among the nonlinear techniques. A common way to approach this problem consists in discretizing the basin using polygons (or other geometries), and iteratively solving the nonlinear inverse problem by local optimization. Nevertheless, this kind of approach is highly dependent of the prior information that is used and lacks from a correct solution appraisal (nonlinear uncertainty analysis). In this paper, we present the application of a full family Particle Swarm Optimizers (PSO) to the 2D gravity inversion and model appraisal (uncertainty assessment) of basement relief in sedimentary basins. The application of these algorithms to synthetic and real cases (a gravimetric profile from Atacama Desert in north Chile) shows that it is possible to perform a fast inversion and uncertainty assessment of the gravimetric model using a sampling while optimizing procedure. Besides, the parameters of these exploratory PSO optimizers are automatically tuned and selected based on stability criteria. We also show that the result is robust to the presence of noise in data. The fact that these algorithms do not require large computational resources makes them very attractive to solve this kind of gravity inversion problems.

1. Introduction

Gravity inversion is a classical tool in applied geophysics (Dobrin, 1960; Nettleton, 1976; Parker, 1994; Telford et al., 1976) to analyze the structure of sedimentary basins in mineral exploration, oil and gas upstream activities, hydrogeology, glaciology, etc. The gravity inverse problem is linear when a given geometry for the bodies under study is provided and the corresponding densities are estimated. Conversely, the inverse problem is nonlinear when the geometry of the bodies is treated as unknown, making some assumptions about the values of the corresponding densities.

Among the nonlinear techniques, the inversion of basement relief of a sedimentary basin is a relatively common task (Barbosa et al., 1997; Blakely, 1995; Chakravarthi and Sundararajan, 2007). The classical way to deal with this problem is via nonlinear optimization, where the unknowns are the depth of the basement at certain locations, or the depth and some additional parameters to take into account the density variation of the sediments. Among others, basement relief estimation has important implications in oil and gas exploration to find out the location of possible stratigraphic traps (Silva et al., 2010), in hydrogeology studies to understand the geological structure of aquifers (Adema et al., 2007; Bohidar et al., 2001), in glaciology to infer the flow rate of discharge (Krimmel, 1970; Stern, 1978; Venteris and Miller, 1993), or in landfill analysis as a tool for density determination (Mantlík et al., 2009) and bottom relief estimation (Silva et al., 2009).

Basement relief estimation based on gravity anomalies could be high dimensional nonlinear inverse problem depending on the model parameterization. Over the years, several methods were used to approach this problem. Some of them were based on the manual modeling of the basin taken into account the adjustment of the observed anomaly by the basin gravimetric model (Bott, 1960). Local optimization techniques, particularly the Levenberg–Marquardt algorithm with Tikhonov's regularization, are commonly used in the resolution of nonlinear inverse problems: stating a prior model for the density distribution (fixed or variable with depth), and using an adequate set of constraints, a solution is achieved through iterative linearization of the cost function (Barbosa et al., 1997; Chakravarthi, 1995; Silva et al., 2006). This procedure provides a solution that is highly dependent on the initial model and on the prior information that are used. Besides, no model appraisal in the nonlinear sense is usually performed on the solution that has been found.

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Uncertainty assessment is a very important step in inversion (see for instance Scales and Snieder, 2000) since inverse problems always belong to decision-making processes, and they are by nature ill-posed, that is, there exist different solutions (called equivalent) that are compatible with the prior information and fit the observed data within the same error bounds. Fernández-Martínez et al. (2012a) studied the cost function landscape for linear and nonlinear inverse problems, showing that equivalent models (for a certain error tolerance) are located within the hyper-quadric region of equivalence in the linear case. This flat elongated valley bends for nonlinear inverse problems. Also, several disconnected basins might compose the nonlinear equivalence landscape.

Uncertainty analysis consists in obtaining a representative set of model parameters in the low misfit region(s) of the cost function topography. Fernández-Martínez et al. (2013) provided a deterministic analysis of the inverse problem uncertainty, proving that the regularization does not provoke the disappearance of the equivalent models, showing the risks of adopting a wrong prior model, and highlighting the fact that linear analysis never accounts for the real uncertainty in nonlinear inverse problems.

Finally, noise in data is an important ingredient in inversion that deforms the cost function topography and it is intimately related to the uncertainty analysis of the solution and to the need of regularization. Fernández-Martínez et al. (2014a,b) have analyzed the effect of noise in inversion, showing that noise deforms the topography of the cost function in a homogeneous/inhomogeneous way, depending if the inverse problem is linear or nonlinear. As a consequence of noise the inverted model never coincides with the hypothetical true model that has generated the observed data. Nevertheless, the true model belongs to the region of equivalence having a higher misfit than the *true* solution. In conclusion, inversion and uncertainty analysis (solution appraisal) must always go hand in hand.

In this paper, a full family of Particle Swarm Optimizers is applied to the gravity inversion and uncertainty assessment of basement relief in sedimentary basins. Particle Swarm Optimization (PSO) is an interesting global optimization technique that was heuristically inspired by the social behavior of groups of animals (birds and fishes) in nature (Kennedy and Eberhart, 1995). Nevertheless, PSO was physically interpreted as a damped-mass spring system (Fernández-Martínez and García-Gonzalo, 2011b). The interest of having at disposal the PSO physical model consists in properly understanding the exploratory behavior of the different PSO family members, and relating stochastic stability of the particle trajectories with the algorithm convergence for any arbitrary statistical distribution of the PSO parameters (García-Gonzalo and Fernández-Martínez, 2014). Although no theoretical results exist to state the theoretic conditions needed for these algorithms to perform a correct posterior sampling, exploratory PSO versions have shown to perform a fairly good sampling of the nonlinear equivalent region when applied to different types of DC electrical inverse problems (Fernández-Martínez et al., 2010a,b), and also in oil and gas production history matching (Fernández-Martínez et al., 2012b). Similar results were also achieved for the Vertical Electrical Sounding inverse problem using binary genetic algorithms (Fernández-Álvarez et al., 2008). These results emphasize the fact that exploration is a key factor in performing a good approximate uncertainty analysis of the inverse problem solution (sampling while optimizing).

In this work we introduce for the first time the use of two novel PSO family optimizers (PP-PSO and RR-PSO) in geophysics, and particularly in 2D basement relief inversion, showing the application to synthetic and real data (a gravimetric profile acquired in the Atacama Desert, north Chile). We also compare the posterior sampling of the nonlinear region of equivalence performed by different PSO family members, analyzing numerically how noise in data affects the topography landscape and, therefore, the corresponding uncertainty analysis of the solution. The uncertainty analysis provided by PSO serves to answer important questions about the basin structure in a probabilistic framework, taking into account the intrinsic uncertainty of the basement relief inverse problem.

2. Classical modeling in 2D basement relief gravity inversion

Gravity inversion of basement relief in sedimentary basins can be proposed as a 2D or 3D problem depending on the model conceptualization. The 2D case is very common, and consists in the inversion of one or different profiles across the basin, generally containing its maximum depth. When the basin is elongated, a common approach consists in inverting several profiles that are orthogonal to its longest dimension, which analyzed together serve to depict a pseudo 3D model. The 2D approach assumes that the anomalous body (in this case the sediments) has an infinity extent in the perpendicular direction to the gravimetric profile.

Several ways to model a generic basin in 2D have been proposed in the literature: modeling the entire basin as a trapezoid (Rao, 1990), using polygons with an arbitrary number of vertices (Chakravarthi et al., 2001), using polynomial functions (Martín Atienza, 2001), or by accretion of rectangular prisms of known density (see for example Chakravarthi, 1995; Barbosa et al., 1997; Silva et al., 2006). Fig. 1 shows a 2D model divided into M prisms, whose density is imposed. The width of the prisms can be selected as a factor (generally between 0.5× and 1.5×) of the average spacing between the N observation points, which do not necessarily need to be equally spaced. Obviously depending on the number of observation points and the number of prisms, the corresponding inverse problem might have an overdetermined or under-determined character, that will highly impact the corresponding uncertainty analysis of the solution (Fernández-Martínez et al., 2012a).

2.1. Modeling of the density contrast

Multiple approximations have been proposed in the literature to model the density contrast $\Delta\rho = \rho_s - \rho_b$ between the sediments and the basement. For environments such as glaciers it is very common to use a constant density value (Krimmel, 1970; Stern, 1978; Venteris and Miller, 1993), although variable depth-density models could be also used (Shumskiy, 1960). For sedimentary basins, a constant density contrast is usually employed (Barbosa et al., 1997; Gabalda et al., 2005). Nevertheless, adopting a variable dependency with depth is also a common strategy, especially for deep environments. Some models for variable density contrast with depth can be seen in Table 1. The parameter needed to define these models of density variation can be estimated at inversion or being adjusted based on borehole information, which is a more realistic approach. The corresponding forward problem equations of gravity attraction following the different density models are presented in the respective papers (Chakravarthi, 1995; Rao, 1990; Rao et al., 1995). The useful exponential depth density variation model introduced in Cordell (1973) has no analytical expression in the space domain, even in the case of simple geometric bodies. Solutions to this problem have been proposed in the frequency domain (Chapell and Kusznir, 2008; Cordell, 1973; Granser, 1987).

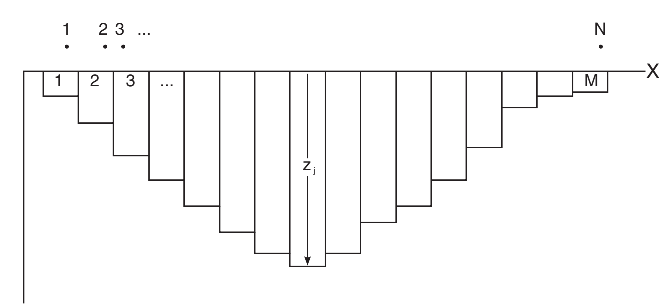

Fig. 1. Two dimensional modeling of a sedimentary basin by right rectangular prisms' accretion. The domain is divided into M rectangular prisms of known density, where depths z_j are unknown. The width of the prisms is a factor of the average separation between the N observed points.