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Effective modeling of ground penetrating radar in fractured media using analytic solutions for propagation, thin-bed interaction and dipolar scattering

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ABSTRACT

We propose a new approach to model ground penetrating radar signals that propagate through a homogeneous and isotropic medium, and are scattered at thin planar fractures of arbitrary dip, azimuth, thickness and material filling. We use analytical expressions for the Maxwell equations in a homogeneous space to describe the propagation of the signal in the rock matrix, and account for frequency-dependent dispersion and attenuation through the empirical Jonscher formulation. We discretize fractures into elements that are linearly polarized by the incoming electric field that arrives from the source to each element, locally, as a plane wave. To model the effective source wavelet we use a generalized Gamma distribution to define the antenna dipole moment. We combine microscopic and macroscopic Maxwell's equations to derive an analytic expression for the response of each element, which describes the full electric dipole radiation patterns along with effective reflection coefficients of thin layers. Our results compare favorably with finite-difference time-domain modeling in the case of constant electrical parameters of the rock-matrix and fracture filling. Compared with traditional finite-difference time-domain modeling, the proposed approach is faster and more flexible in terms of fracture orientations. A comparison with published laboratory results suggests that the modeling approach can reproduce the main characteristics of the reflected wavelet.

1. Introduction

Identification and characterization of permeable fractures within rock formations are of central interest in hydrology (National Research Council, 1996). The flow and transport behavior in fractured media can be very complex and difficult to infer from traditional hydrological experiments (Neuman, 2005). A promising approach is to combine hydrologic measurements with ground penetrating radar (GPR) data (e.g. Olsson et al., 1992). Both surface reflection and cross-borehole tomographic monitoring studies have been used to infer the spatial distribution of tracer plumes and to dynamically image tracer transport through preferential flow paths (Birken and Versteeg, 2000; Tsoflias et al., 2001; Day-Lewis et al., 2003; Talley et al., 2005; Becker and Tsoflias, 2010; Dorn et al., 2011, 2012a). Furthermore, the ability of GPR to provide information about mm-thick fractures has been demonstrated theoretically (Hollender and Tillard, 1998; Bradford and Deeds, 2006; Tsoflias and Hoch, 2006), through controlled experiments (Grégoire and Hollender, 2004; Tsoflias et al., 2004; Sambuelli and Calzoni, 2010) and by field-based investigations (Tsoflias and Hoch, 2006; Sassen and Everett, 2009; Dorn et al., 2011, 2012b). In the complex environment found in most fractured rock systems, efficient and effective interpretation of GPR measurements must rely on forward models that accurately simulate the experiments.

When an electromagnetic wave impinges on a thin layer, a series of complex interference phenomena occur that alter both the phase and amplitude of the reflected and transmitted waves. Such phenomena have been studied extensively in optics and exact solutions are available by applying the macroscopic Maxwell's equations and associated boundary conditions on the sides of a dielectric slab (e.g., a fluid filled fracture). These solutions have been used in geophysics to describe how the GPR signal reflected from fractures varies as a function of material properties, fracture thickness (aperture) and orientation (Tsoflias and Hoch, 2006; Tsoflias and Becker, 2008).

Numerical GPR forward modeling schemes do not incorporate the analytic nature of the effective reflection coefficients since space discretization and medium parameterization implicitly account for boundaries, across which the macroscopic Maxwell's equations are solved. As spatial discretization becomes finer, the macroscopic numerical solutions approach the analytically derived Fresnel reflection and transmission coefficients. However, the finite spatial discretization gives rise to errors, especially when modeling irregular geometries or fine-scale structures. Sub-discretization schemes have been recently

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proposed (e.g., Diamanti and Giannopoulos, 2009) but the computational demand still remains for 3D implementations. Moreover, irregular geometries still pose a problem since FDTD codes usually implement a Cartesian grid and tilted planar surfaces are not discretized exactly; a known problem that is often referred to as "staircasing". In numerical solvers based in the time domain, insufficient temporal sampling can also give rise to numerical dispersion (Bergmann et al., 1998). Ray-tracing algorithms can include effective reflection coefficients, but they rely on the plane wave assumption being valid everywhere along an interface and only consider the far-field region of electromagnetic radiation. Furthermore, ray-tracing workflows are often based on algorithms developed for seismic processing (Dorn et al., 2012b) and ignore the polarized response of GPR sources and reflections.

A more general approach is to consider a fracture as a polarizable dielectric and conductive anomaly, in which many infinitesimal dipoles are induced and oscillate in response to the incident field. This approach is exactly described by the microscopic Maxwell's equations (e.g. Purcell and Smith, 1986), in which matter is seen as a collection of polarizable particles. The macroscopic boundary conditions can then be derived as limiting cases of the microscopic approach through the Ewald–Oseen extinction theorem (Fearn et al., 1996). The macroscopic approach is thus an averaged version of the microscopic formulation, the latter not only being correct in the quantum regime but also more intuitive (Feynman et al., 1969). A numerical modeling application of the microscopic Maxwell's equations has been used extensively by the astrophysical community to describe light scattering from dielectric objects — see Yurkin and Hoekstra (2007) for an overview — but we are not aware of applications to GPR scattering.

We propose a forward modeling approach that uses analytic solutions to simulate the propagation of electromagnetic waves within homogeneous media and the scattering of the waves from fractures. The fractures are seen as dielectric and conductive anomalies that are polarized by the incident EM field and are defined as rectangular planes with a given midpoint, azimuth, dip, thickness and material filling. Each fracture plane is discretized into polarizable elements, a formulation which enables simulating heterogeneous tracer concentrations in the fractures by varying the electrical properties of each element over time, and also accounts for the change in direction and magnitude of the incident electric field along the fracture plane. The elements are modeled as infinitesimal dipoles that are polarized linearly and in parallel to the incident electric field. The main difference from the astrophysical formulation is that we only assign effective dipoles along the plane of the fracture. To account for the effect of the dipoles along the direction normal to the fracture plane we apply the Ewald–Oseen extinction theorem and scale the dipoles by the effective reflection coefficients of a thin layer. Another difference is that we only consider the incident field caused by the external source and do not account for interactions between elements. We use analytical expressions of the Maxwell equations in a homogeneous space to describe the propagation of the EM field to and from each element and allow for frequency-dependent attenuation and dispersion through the Jonscher constitutive formulation (Jonscher, 1999). The resulting forward modeling scheme is free from boundary effects related to the modeled domain size and also from discretization errors. We begin by describing the theory before proceeding with how we discretize a fracture, and, finally, we compare our forward modeling scheme to simulations based on a well-established numerical code and to laboratory data.

2. Theory

The electromagnetic properties of matter that characterize the velocity, attenuation and dispersion of electromagnetic (EM) energy in dielectric media are the magnetic permeability μ (N A^{-2}), the electric permittivity ε (F m^{-1}) and the conductivity σ (S m^{-1}), or equivalently the resistivity, ρ (Ω m), with $\rho = \sigma^{-1}$. These parameters are in general complex-valued and frequency dependent, while for many practical geophysical purposes it is safe to assume the magnetic permeability to be constant and equal to the value in vacuum, $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}. Reflections and transmissions arise at the boundary between contrasting media and are a form of energy scattering. For geophysical purposes it is customary to use the macroscopic Maxwell equations as the governing physical principles to describe such systems (Zonge et al., 1991) and the link between the propagating field to a given medium is made through the constitutive relations, $\mathbf{D} = \varepsilon\,\mathbf{E}$ and $\mathbf{J} = \sigma\,\mathbf{E}$, where $\mathbf{E}$ (V m^{-1}) is the incident electric field arising from a distant source, $\mathbf{J}$ (A m^{-2}) is the resulting current density and $\mathbf{D}$ (C m^{-2}) is the electric displacement field.

There is a theoretical distinction between permittivity and conductivity because the first describes polarization effects resulting from bound charge and the second conduction effects resulting from free charge. In practice, these two parameters can be combined since one can only measure the in-phase and out-of-phase components of the current (Hollender and Tillard, 1998). It is thus convenient to define the effective permittivity ε_e (F m^{-1}) with real and imaginary parts that characterize the propagation properties of the material: wave velocity, attenuation and dispersion.

2.1. The microscopic viewpoint

While the electric displacement field $\mathbf{D}$ was introduced by Maxwell and is proportional to the "bound" charge density within a dielectric (Purcell and Smith, 1986), it is only an approximation resulting from spatial averaging of a microscopic process that involves interaction between fields and particles that make up matter. The microscopic description was introduced by Lorentz (1916) and considers a dielectric as a collection of particles that undergo electronic polarization from an externally applied electric field. The applied field exists independently of the dielectric medium and travels through the dielectric medium at the speed of light in vacuum, that is, in the free space between the particles of the dielectric. As it travels through the medium it polarizes the particles that make up the dielectric, inducing moments of charge distribution in each particle. For neutral dielectrics it is only the electric dipole moment that needs to be considered and polarization can be seen as the result of an induced charge separation that generates an electric dipole moment $\mathbf{p} = q\ dL\ \hat{\mathbf{r}}_d$ for separation dL (m) between two opposite charges of equal magnitude q (C) and orientation $\hat{\mathbf{r}}_d$. The electric field produced by such a dipole moment can be accurately calculated for an observation distance r (m) much larger than the charge separation dL producing the dipole moment and becomes exact in the limiting case, $\frac{dL}{r} \to 0$, in which the induced dipole is often called a point dipole. The electric field of the point dipole is given by:

$$\mathbf{E}_d(\mathbf{r},\mathbf{p}) = \frac{1}{4\pi\varepsilon_0}\left\{k^2(\hat{\mathbf{r}}\times\mathbf{p})\times\hat{\mathbf{r}} + (3\ \hat{\mathbf{r}}\ (\hat{\mathbf{r}}\cdot\mathbf{p}) - \mathbf{p})\left(\frac{1}{r^2} - \frac{i\ k}{r}\right)\right\}\frac{e^{i\,k\,r}}{r} \quad (1)$$

where $\hat{\mathbf{r}}$ is a unit vector pointing from the point dipole to the point of observation $\mathbf{r} = r\ \hat{\mathbf{r}}$, r (m) is the magnitude of $\mathbf{r}$, $\varepsilon_0 \equiv 8.854\ldots \times 10^{-12}$ F m^{-1} is the electric permittivity in vacuum and k (rad m^{-1}) is the wavenumber in vacuum. The vacuum wavenumber is given by $k = \omega\,c^{-1}$, where ω (rad s^{-1}) is the angular frequency and $c \equiv 299\,792\,458$ m s^{-1} is the speed of light in vacuum. Eq. (1) includes the near, intermediate and far-fields generated by a dipole $\mathbf{p}$ located at the origin of the coordinate system. We use the subscript d in the electric field ($\mathbf{E}_d$) to denote that it corresponds to a point dipole. A generalized expression for the electric field at an arbitrary location $\mathbf{r}$ generated from a dipole located at $\mathbf{r}'$ is easily obtained through the substitution $\mathbf{r} \to \mathbf{r}-\mathbf{r}'$. A detailed derivation of Eq. (1) can be found in classical electrodynamics textbooks (e.g., Jackson, 1998).

Each particle in a dielectric medium is polarized by a superposition of the applied field generated by a source far away and the fields generated by all the other particles present in the dielectric. For a uniform