Journal of Applied Geophysics 116 (2015) 236–246

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

A stable and self-adaptive approach for inverse Q-filter

Gulan Zhang [a,b,*], Ximing Wang [b], Zhenhua He [a]

[a] Chengdu University of Technology, Chengdu, Sichuan, China
[b] BGP, CNPC, Zhuozhou, Hebei, China

ARTICLE INFO

Article history:
Received 28 October 2014
Received in revised form 4 March 2015
Accepted 5 March 2015
Available online 10 March 2015

Keywords:
Inverse Q-filter
Gain-limit
Dynamic range
Self-adaptive
Effective frequency band
Stabilizing factor

ABSTRACT

The inverse Q-filter procedure attempts to eliminate the effect of the Earth Q-filter and hence improve the seismic resolution. The numerical instability of inverse Q-filter amplitude compensation reduces the SNR (signal-to-noise ratio) and limits the spatial resolution. Although the gain-limit constrained stable factor method can control the numerical instability and the SNR, but its gain-limit is time-invariant and is not associated with the seismic data; then it usually suppresses high frequencies at later times and reduces the seismic resolution. In this paper, we focus on understanding the impact of the gain-limit, the Q value and the dynamic range of seismic data to the seismic resolution, and propose a self-adaptive method for inverse Q-filter amplitude compensation. The gain-limit in the self-adaptive method is time-variant and self-adaptive to the cut-off frequency of the effective frequency band of seismic data; and the stabilizing factor changes in inverse proportion to the square of the self-adaptive gain-limit; then, the self-adaptive method can restore energy in the effective frequency band and control the numerical instability, and finally achieve high resolution and high SNR seismic data. Synthetic and real data examples demonstrate that the self-adaptive inverse Q-filter compensates for energy loss without boosting high frequency noise, and produces desirable seismic images with high resolution and high SNR.

1. Introduction

The Earth Q-filter, with frequency-dependent amplitude attenuation and velocity dispersion (Futterman, 1962; Kjartansson, 1979; Wang and Guo, 2004a), distorts seismic wavelet and reduces the seismic resolution. Theoretically, the seismic resolution can be greatly enhanced after eliminating the influence from the time-variant wavelet, if we could accurately estimate the Q factor from either VSP data (Hauge, 1981; Raikes and White, 1984; Stainsby and Worthington, 1985; Tonn, 1989, 1991; Pujol et al., 1998; Blias, 2012; Zhang et al., 2014; Wang, 2014) or surface seismic data (Clark et al., 2001; Wang, 2004) and further use it for inverse Q-filtering (Hale, 1981, 1982; Hargreaves and Calvert, 1991; Bano, 1996; Wang, 2002, 2003, 2006) during seismic data processing and migration (Wang and Guo, 2004b; Wang, 2008).

The full inverse Q-filter contains the phase correction and the amplitude compensation. Furthermore, the phase correction can correct for the phase distortion from velocity dispersion, while amplitude compensation can compensate for the attenuated amplitude, and then recover the seismic resolution. As inverse Q-filter is the reverse process of forward wave propagation (Robinson, 1979), thus it can be accomplished by a method similar to seismic deconvolution (Hale, 1981, 1982; Bickel and Natarajan, 1985) or the Stolt frequency wave number migration (Hargreaves and Calvert, 1991).

Efficiency and stability are two general concerns of the inverse Q-filter. Considering computational efficiency, Hargreaves and Calvert (1991) and Bano (1996) propose the phase-only inverse Q-filter methods for correcting the phase distortion from velocity dispersion and these methods are unconditionally stable (Robinson, 1979, 1982; Bickel and Natarajan, 1985). Whereas, the amplitude compensation is essential to enhance the seismic resolution, and it should not be ignored and should be carefully handled because the amplitude compensation operator is an exponential function of the frequency; therefore, it may cause numerical instability and generate undesirable artifacts in the seismic data.

In order to control the numerical instability and the SNR (signal-to-noise ratio), Bickel and Natarajan (1985) proposed the gain-limit (the maximum value of inverse Q-filter amplitude compensation) constrained cut-off frequency method. For further control the numerical instability and the SNR, James and Knight (2003) and Wang (2006) proposed the gain-limit constrained stable factor methods. The inverse Q-filter amplitude compensation functions for the gain-limit constrained stable factor methods are presented in the form of fraction, while James and Knight (2003) added a stabilizing factor to the denominator, and Wang (2006) added the stabilizing factor to both the denominator and numerator. But the gain-limit in these methods are all time-invariant; when the gain-limit is too small, these methods can control the numerical instability

* Corresponding author at: BGP, CNPC, Zhuozhou, Hebei, China.
E-mail address: zhanggulan@cnpc.com.cn (G. Zhang).

http://dx.doi.org/10.1016/j.jappgeo.2015.03.012
0926-9851/

and SNR, but the seismic resolution decreases at later times; when the gain-limit is too large, the seismic resolution improves but at the expense of the SNR.

In this paper, we first explain the numerical instability of the inverse *Q*-filter based on the theory of inverse *Q*-filter amplitude compensation and the dynamic range of the seismic data. Then, we describe the stable factor inverse *Q*-filter method of James and Knight (2003) and explicitly establish the relationship between the gain-limit and the stabilizing factor in the stable factor method of James and Knight (2003). After that, we give the gain-limit selection criterion for seismic resolution enhancement and use synthetic data tests to verify it. Then, we introduce a novel self-adaptive method for inverse *Q*-filter amplitude compensation based on the stable factor method of James and Knight (2003) and the gain-limit selection criterion. The gain-limit in the self-adaptive method is time-variant and self-adaptive to the cut-off frequency of the effective frequency band of the seismic data, and the stabilizing factor changes in inverse proportion to the square of the self-adaptive gain-limit. Finally, we test the self-adaptive method on synthetic and real data.

2. Inverse *Q*-filter

2.1. Basics of the inverse *Q*-filter

The Earth *Q*-filter can be based on the 1-D wave equation.

$$\frac{\partial^2 U(r,\omega)}{\partial r^2} + k^2 U(r,\omega) = 0, \tag{1}$$

where $U(r, \omega)$ is the plane wave of the radial frequency ω at travel distance r, and k is the wave number.

Eq. (1) has an analytic solution

$$U(r+\Delta r,\omega) = U(r,\omega)\exp(-jk\Delta r), \tag{2}$$

where $j = \sqrt{-1}$. The distance increment Δr can be replaced by

$$\Delta r = v(\omega_0)\Delta t, \tag{3}$$

where $v(\omega_0)$ is the phase velocity of the reference frequency ω_0, and Δt is the travel time increment. The Earth *Q*-filter effect is introduced in the definition of the complex-valued wave number k

$$k = \frac{\omega}{v(\omega)}\left(1 - \frac{j}{2Q}\right), \tag{4}$$

where $v(\omega)$ is the frequency-dependent phase velocity, Q is the quality factor of the medium.

$$v(\omega) = v(\omega_0)\left|\frac{\omega}{\omega_0}\right|^{\frac{1}{\pi Q}}. \tag{5}$$

Substituting wave number k into Eq. (2), we have

$$U(t+\Delta t,\omega) = U(t,\omega)\exp\left(-\frac{\omega}{2Q}\Delta t\right)\exp\left(-j\omega\left|\frac{\omega}{\omega_0}\right|^{-\frac{1}{\pi Q}}\Delta t\right), \tag{6}$$

and the basis for the inverse *Q*-filter

$$U(t+\Delta t,\omega) = U(t,\omega)\exp\left(\frac{\omega}{2Q}\Delta t\right)\exp\left(j\omega\left|\frac{\omega}{\omega_0}\right|^{-\frac{1}{\pi Q}}\Delta t\right). \tag{7}$$

Then, the inverse *Q*-filter amplitude compensation function can be written as

$$B(t,\omega,Q) = \exp\left(\frac{\omega}{2Q}t\right). \tag{8}$$

2.2. Numerical instability of inverse *Q*-filter

To demonstrate the numerical instability of the inverse *Q*-filter, we consider a simple synthetic example shown in Fig. 1; Fig. 1a shows a synthetic trace; Fig. 1b shows the effect of the Earth *Q*-filter with constant Q ($Q = 100$) to synthetic trace displayed in Fig. 1a; Fig. 1c shows the result of the inverse *Q*-filter to the synthetic signal displayed in Fig. 1b. The inverse *Q*-filter process corrects the phase and restores the amplitude. However, there are strong artifacts when the time is greater than 1 s, even though the input signal is noise free.

The appearance of noise in the output signal is a consequence of the basic inverse *Q*-filter procedure . The source of the noise is mainly from three aspects. First, the seismic wave is attenuated gradually during the Earth *Q*-filter, and beyond a certain time, the amplitude would be too weak and below the ambient noise level. Second, the background noise is mainly the numerical error associated with the maximum dynamic range of equipment (e.g., geophone and computer). Thus, when the amplitude is below the background noise, the seismic signal is buried under the background noise; Third, the truncation error of the signal processing (e.g., Fourier transform and inverse Fourier transform) mainly appears as terminal artifacts (the undesirable artifacts at the terminal of the seismic signal). Unfortunately, the inverse *Q*-filter amplitude compensation not only recovers the signal, but also amplifies the ambient noise, the background noise and the terminal artifacts. In the case of noise-free synthetic data, the background noise is the equipment errors relative to the storage precision. The strong artifacts are considered to be the numerical instability of inverse *Q*-filter.

3. The traditional stable factor method for inverse *Q*-filter

The amplitude compensation function is an exponential function of the frequency, and it may cause numerical instability and generate undesirable artifacts in the seismic data. In order to control the numerical instability and SNR, James and Knight (2003) changed the amplitude compensation function into a fractional form, and added a stabilizing factor to the denominator. Here, we consider the stable factor inverse *Q*-filter method of James and Knight (2003) as the traditional stable factor method.

3.1. The traditional stable factor method for inverse *Q*-filter amplitude compensation

The amplitude compensation function for the traditional stable factor method (James and Knight, 2003) can be expressed as

$$S_1(t,\omega,Q(t)) = \frac{\dfrac{1}{B(t,\omega,Q(t))}}{\dfrac{1}{B^2(t,\omega,Q(t))}+\beta}, \tag{9}$$

where $S_1(t, \omega, Q(t))$ is the amplitude compensation function, $Q(t)$ is the medium quality factor at travel time t, β is the stabilizing factor (a small positive number), and $B(t, \omega, Q(t))$ is the amplitude compensation function based on Eq. (8) as

$$B(t,\omega,Q(t)) = \exp\left(\frac{\omega}{2Q(t)}t\right). \tag{10}$$