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A numerical study on the relation between the electrical spectra of a mixture and the electrical properties of the components of the mixture

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ABSTRACT

To understand the effect of the components' electrical properties on the electrical spectra of a mixture, a three-dimensional finite difference method (3D-FDM) is employed to extract the effective permittivity and conductivity of mixtures. The simulation results indicate that: (1) the variation of the conductivity of the inclusion and the host could control the effective permittivity of the mixtures at the low frequency side; (2) the variation of the permittivity of the host and the inclusion could control the effective conductivity of the mixtures at the high frequency side; (3) with the increase of the permittivity of the inclusion and the host whose permittivity has a fixed ratio,ttra the spectral curve of the effective permittivity shifts to the upper-left corner and that of the effective conductivity shifts to the left; (4) with the increase of the conductivity of the inclusion and the host whose conductivity has a fixed ratio, the spectral curve of the effective permittivity shifts to the right and that of the effective conductivity shifts to the upper-right corner. Therefore, this study revealed some intrinsic physical quality of the electrical spectra of mixtures and provided a new idea for the explanation of electrical spectra dispersion of rocks.

1. Introduction

Sandstone, limestone, shale and soil are typical mixtures from a subsurface formation. They are usually composed of several components with different electrical properties. Therefore, their macro-electrical parameters are affected by the electrical parameters of the components they contain. Many experiments indicate that the effective conductivity and permittivity of mixtures are frequency-dependent (Jonscher, 1977; Kruschwitz, 2008; Lesmes and Frye, 2001; Lesmes and Morgan, 2001; Scott, 2003; Slater et al., 2006; Sturrock, 1999). The electrical properties of the earth materials are of great significance for environment monitoring (Robert and Lin, 1997; Slater and Lesmes, 2002), reservoir parameter evaluation (Weller et al., 2010a,2010b), hydraulic property assessment (Binley et al., 2005; Slater, 2007), earthquake prediction (Mogi, 1985) and so on.

In order to explain the frequency dispersion of conductivity and permittivity of rocks, some empirical models were developed, such as the Debye model (Debye, 1929), the Cole–Cole model (Cole and Cole, 1941; Pelton et al., 1978) and the Dias model (Dias, 2000). The Cole–Cole model uses some empirical coefficients to control the shape and position of the spectroscopy curves and has been successful at fitting data from most experiments. However, these empirical models can only fit the dispersion curves of permittivity and conductivity but cannot reveal the intrinsic physics of characteristics of frequency dispersion.

Based on effective media theory, some analytic equations were developed to calculate the effective permittivity and conductivity of simple composites which consist of spherical or ellipsoidal inclusions (Liu and Shen, 1993; Maxwell, 1904; Reynolds and Hough, 1957). However, the analytic methods can only be used to compute the composites with very simple structures.

In order to study the composites with more complex structure, scientists introduced a variety of numerical methods to study the effective permittivity of mixtures with periodic, stratified or random structures (Boudida et al., 1998; Brosseau, 2006; Beroual and Brosseau, 2001; Brosseau and Beroual, 2001; Mejdoubi and Brosseau, 2006a; Mejdoubi and Brosseau, 2006b; Myroshnychenko and Brosseau, 2005a; Myroshnychenko and Brosseau, 2005b; Sareni et al., 1996a; Sareni et al., 1996b; Sareni et al., 1997a; Sareni et al., 1997b). Some of the numerical methods can also be used to obtain the complex permittivity of the mixtures (Myroshnychenko and Brosseau, 2005a, 2008; Sareni et al., 1996a).

The effect of shape (Beroual et al., 2000; Brosseau and Beroual, 2003; Mejdoubi and Brosseau, 2006c; Qin and Brosseau, 2012), spatial orientation (Beroual et al., 2000), and volume ratio (Beroual et al., 2000; Myroshnychenko and Brosseau, 2008) of the inclusion on the effective permittivity of mixtures can also be analyzed conveniently by these numerical methods.

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So far, a number of numerical methods have been successfully used to calculate the effective permittivity and conductivity of complicated composites, such as the finite difference method (FDM) (Kärkkäinen et al., 2001), finite element method (FEM) (Krakovský and Myroshnychenko, 2002; Mejdoubi and Brosseau, 2006a,2006c; Myroshnychenko and Brosseau, 2005a,2005b; Sareni et al., 1996a; Zhao et al., 2004), boundary-integral equation method (BIEM) (Ghosh and Azimi, 1994; Sareni et al., 1997a,1997b), boundary element method (BEM) (Sekine et al., 2002) and finite-difference time-domain (FDTD) method (Mejdoubi and Brosseau, 2006d; Wu et al., 2007).

In addition, the digital core modeling (DCM) technique developed rapidly in recent years. The DCM technique can be used to simulate mixtures with non-periodic structure (Zhu et al., 2012). Liu et al. (2009) discussed the application of the DCM technique in the simulation of electrical properties of rocks and showed some advantages of the DCM technique.

Numerical modeling can overcome the bias caused by laboratory measurements and obtain effective dielectric constant of rocks with given geometric structures and physical properties. In this study, we compute the effective permittivity and conductivity of models of simple composites by a 3D-FDM. The purpose of this study is to reveal the relation between the characteristics of spectroscopy of a mixture and the electrical parameters of the component that the mixture contains.

2. Methodology

Taking sand stone as an example, it can be assumed to be a mixture with periodic structure. According to the mixing law, we can use one unit of a mixture with periodic structure to evaluate the electrical properties of the mixtures (Fig. 1). Based on the quasielectrostatic assumption, a 3D-FDM can be used to extract the effective dielectric constant and conductivity of the 3D model of mixtures with periodic structure (Asami, 2006).

The 3D-FDM method uses a cubic capacitor model which has $n \times n \times n$ cubic element cells (Fig. 2(a)) and $n \times n \times n$ nodes. The node is located at the center of each cubic element (Fig. 2(b)). In the cubic model, we can have any symmetrical inclusion which can make the model satisfy the periodic boundary conditions. Each node is connected with the other six nodes (Fig. 2(b)). Since there is neither current sink nor source in the element (i, j, k), the total electric currents that flow into the cubic element equal to zero,

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = 0, \tag{1}$$

where I_1 is the current that flows into the node (i, j, k) from the node $(i-1, j, k)$.

$$I_1 = \left(\phi_{i-1,j,k} - \phi_{i,j,k}\right) Y_1, \tag{2}$$

where Y_1 is the admittance between the node $(i-1, j, k)$ and the node (i, j, k) (Fig. 3(d)) and $\phi(i, j, k)$ is the potential on the node (i, j, k). Therefore, Y_1 is the series admittance of a half of the element cube $(i-1, j, k)$ and a half of the element cube (i, j, k),

$$\frac{1}{Y_1} = \frac{1}{2Y_{i-1,j,k}} + \frac{1}{2Y_{i,j,k}}, \tag{3}$$

$$Y_1 = \frac{2d\sigma^*_{i-1,j,k}\sigma^*_{i,j,k}}{\sigma^*_{i-1,j,k} + \sigma^*_{i,j,k}}. \tag{4}$$

In the same way, we can get the equations of I_2, I_3, I_4, I_5 and I_6. Then the finite difference equation of $\phi_{i,j,k}$ can be derived from Eq. (1) and written as:

$$\begin{aligned}\phi_{i,j,k} = {} & (Y_1\phi_{i+1,j,k} + Y_2\phi_{i-1,j,k} + Y_3\phi_{i,j+1,k} + Y_4\phi_{i,j-1,k} + Y_5\phi_{i,j,k+1} \\ & + Y_6\phi_{i,j,k-1})(Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6)^{-1}.\end{aligned} \tag{5}$$

The potentials on the top and the bottom face are fixed at 0 and 1 V, respectively. The initial potentials of the other nodes are set to be 0.5 V. On the other four sides of the cube, there is no current pass through them since there is no source applied in these directions. The nodal potentials $\phi(i, j, k)$ are unknown, we can establish $(n-2) \times n^2$ simultaneous equations for the potentials at all the nodes except those on the top and the bottom side. The simultaneous equations are solved by the successive over-relaxation iteration algorithm. By summing up currents on the top surface, we can obtain the total current that goes through an x–y plane,

$$I = \sum_{i=1}^{n}\sum_{j=1}^{n} I_{i,j} = \sum_{j=1}^{n}\sum_{i=1}^{n} Y_{i,j}\left(\phi_{i,j,k+1} - \phi_{i,j,k}\right). \tag{6}$$

By using $Y = I / V$, the effective complex conductivity σ^* of the cubic model can be calculated,

$$\sigma^* = \frac{I}{V}\frac{L}{S} = \sigma_{eff} + j\omega\varepsilon_0\varepsilon_{eff}, \tag{7}$$

where L is the height of the cubic model, $V = V_1$–V_0, S is the area of the top face of the model (Fig. 2(a)), σ_{eff} and $\varepsilon_{\mathrm{eff}}$ are effective conductivity and relative dielectric constant of the cubic system, respectively.

According to the mixing law, we need only to analyze one unit element of the periodic composite material to extract the effective electrical parameters. One unit element usually consists of a host and inclusion (Fig. 4). The electrical parameters in our model are complex, for example, the complex dielectric constant of the host is

$$\varepsilon^*_h = \varepsilon_0\varepsilon_h - j\frac{\sigma_h}{\omega}, \tag{8}$$

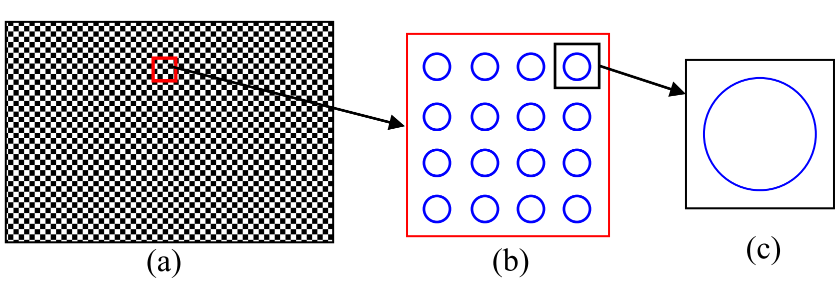

Fig. 1. Simplified model of a mixture with periodic structure. (a) The mixture with periodic structure, (b) a patch of the mixture, (c) a unit of the mixture.