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Frequency and time domain three-dimensional inversion of electromagnetic data for a grounded-wire source

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ABSTRACT

We present frequency- and time-domain three-dimensional (3-D) inversion approaches that can be applied to transient electromagnetic (TEM) data from a grounded-wire source using a PC. In the direct time-domain approach, the forward solution and sensitivity were obtained in the frequency domain using a finite-difference technique, and the frequency response was then Fourier-transformed using a digital filter technique. In the frequency-domain approach, TEM data were Fourier-transformed using a smooth-spectrum inversion method, and the recovered frequency response was then inverted. The synthetic examples show that for the time derivative of magnetic field, frequency-domain inversion of TEM data performs almost as well as time-domain inversion, with a significant reduction in computational time. In our synthetic studies, we also compared the resolution capabilities of the ground and airborne TEM and controlled-source audio-frequency magnetotelluric (CSAMT) data resulting from a common grounded wire. An airborne TEM survey at 200-m elevation achieved a resolution for buried conductors almost comparable to that of the ground TEM method. It is also shown that the inversion of CSAMT data was able to detect a 3-D resistivity structure better than the TEM inversion, suggesting an advantage of electric-field measurements over magnetic-field-only measurements.

1. Introduction

Electromagnetic surveying has been widely used for problems associated with resource exploration, as well as in environmental and geotechnical applications. There are a number of different electromagnetic (EM) methods with different characteristics depending on source and receiver type, measured quantities, and source–receiver geometries. Controlled-source EM techniques employ either grounded wires or loops as the signal source. Grounded wires are more suited for deep surveys, because at large distances, the EM field falls off less rapidly from grounded wires than from loops (Spies and Frischknecht, 1991).

There are two popular EM techniques that utilize a grounded-wire source: the long-offset transient electromagnetic (LOTEM) method (e.g., Strack, 1992) and the controlled-source audio-frequency magnetotelluric (CSAMT) method (e.g., Zonge and Hughes, 1991). In addition, an airborne version of the LOTEM survey, called GREATEM, was recently proposed by Mogi et al. (2009). In these surveys, the source is located outside the area of interest and remains fixed at a single position for the entire survey. One common feature of long-offset configurations is that galvanic currents are dominant (Gunderson et al., 1986), which makes measurements sensitive to lateral variations in electrical conductivity near the receiver and in-between the source and receiver. Thus, three-dimensional (3-D) interpretation is generally required for LOTEM and CSAMT data.

Inversion of transient electromagnetic (TEM) data for 3-D distributions of conductivity (or its reciprocal, resistivity) can be done directly in the time domain. Wang et al. (1994) introduced an imaging algorithm based on the concept of back-propagation and the explicit finite-difference time-domain (FDTD) method. Newman and Commer (2005) advanced and extended it to a 3-D inversion scheme by implementing on a massively parallel computer. In Haber et al. (2004), finite volume methods and an implicit method were adopted for time-domain modeling. In addition, based on the work of Haber et al. (2007), Oldenburg et al. (2013) developed an efficient 3-D inversion method for large-scale problems with multiple sources, using matrix-factorization software available on massively parallel computing platforms. It is also possible to obtain the time-domain solution using discrete inverse Fourier transform of the frequency-domain solution (Mulder et al., 2008; Newman et al., 1986). Cox et al. (2012) used an integral equation technique and cosine transform to compute the time-domain responses and sensitivities. While much progress has been made with all these

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works, the computational demands of 3-D inversion techniques are still too large for routine application.

The purpose of this study is to develop 3-D inversion methods that can be applied to TEM and frequency-domain CSAMT data from grounded sources, using computing resources available to exploration geophysicists. We will first outline a time-domain inversion approach that is based upon a finite-difference (FD) method and inverse Fourier transform of the frequency-domain solution. Using a synthetic example, we will then compare the performances of time-domain inversion for the impulse and step responses from a LOTEM survey. As an alternative to time-domain inversion, we will also test a frequency-domain approach that employs a Fourier transform technique, called smooth-spectrum inversion (Mitsuhata et al., 2001), to recover the frequency response from TEM sounding data and perform inversions in the frequency domain. Finally, we will compare the resolution capabilities of ground-based and airborne LOTEM and CSAMT data from a common grounded source.

2. Forward modeling and sensitivity

In this section, we briefly describe our approach to the solution of the frequency- and time-domain forward problems and sensitivity calculation, which is based on a FD solution in the frequency domain and subsequent inverse Fourier transformation for time-domain problems.

2.1. Frequency-domain FD modeling

If displacement currents can be ignored, the differential equation for the electric field, derived from Maxwell's equations, is given by

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r},\omega) + i\omega\mu_0\sigma(\mathbf{r})\mathbf{E}(\mathbf{r},\omega) = -i\omega\mu_0\mathbf{J}_s(\mathbf{r},\omega), \quad (1)$$

where σ is the conductivity, μ_0 is the magnetic permeability of free space, and $\mathbf{J}_s$ denotes the impressed source current. Here, the electric field $\mathbf{E}$ is a function of the angular frequency ω and position $\mathbf{r}$. Dividing the total field into primary and secondary parts, we obtain an equation for the secondary field:

$$\nabla \times \nabla \times \mathbf{E}_s + i\omega\mu_0\sigma\mathbf{E}_s = -i\omega\mu_0\left(\sigma - \sigma_p\right)\mathbf{E}_p, \quad (2)$$

where σ_p is the conductivity of a (background) uniform or layered half-space and $\mathbf{E}_p$ is its electric field, which can be obtained semi-analytically.

Following Newman and Alumbaugh (1995), we used a staggered grid to discretize 3-D conductivity models. As a boundary condition, the tangential component of the secondary electric field is set to zero at the boundaries. The resultant matrix system is solved using a complex version of the incomplete Cholesky conjugate gradient (ICCG) method, with the exception that the incomplete Cholesky decomposition is applied only to the diagonal sub-blocks that are positive-definite (Mackie et al., 1994). A correction method (Smith, 1996) that enforces a divergence-free condition is also used to accelerate the convergence rate of the iterative solution.

Once the electric field is obtained, the magnetic field above the earth's surface can be calculated through

$$\mathbf{H} = -\frac{1}{i\omega\mu_0}\nabla \times \mathbf{E}. \quad (3)$$

2.2. Frequency-domain sensitivity

Solving nonlinear inverse problems involves calculating the Jacobian matrix. The elements of this matrix are the sensitivity or partial derivatives of the response with respect to the model parameters. The sensitivities of EM field components with respect to conductivity can be efficiently obtained by the adjoint-equation method (e.g., McGillivray et al., 1994).

Denoting the conductivity within a domain, D, as σ_k, the sensitivity for the i (=x, y, z) component of the electric field at a receiving position $\mathbf{r}_0$ is given by

$$\frac{\partial E_i(\mathbf{r}_0,\omega)}{\partial\sigma_k} = \int_D \mathbf{E}(\mathbf{r}',\omega)\cdot\widetilde{\mathbf{E}}^J(\mathbf{r}',\omega)dv', \quad (4)$$

where $\mathbf{E}$ is the electric field within the region of interest produced by the actual source, and $\widetilde{\mathbf{E}}^J$ is the electric field produced by the unit electric dipole placed at $\mathbf{r}_0$ and oriented along the same direction as the receiver. Similarly, the sensitivity for the i (=x, y, z) component of the magnetic field becomes

$$\frac{\partial H_i(\mathbf{r}_0,\omega)}{\partial\sigma_k} = -\frac{1}{i\omega\mu_0}\int_D \mathbf{E}(\mathbf{r}',\omega)\cdot\widetilde{\mathbf{E}}^M(\mathbf{r}',\omega)dv', \quad (5)$$

where $\widetilde{\mathbf{E}}^M$ denotes the electric field produced by the i-directed unit magnetic dipole at $\mathbf{r}_0$.

Eqs. (4) and (5) show that the sensitivities can be obtained by carrying out forward modeling with fictitious and actual sources and integrating the inner product of the two electric fields over the region of interest. Note that the number of additional forward modeling steps required to calculate the Jacobian matrix is equal to the number of receiving positions rather than the number of discretized conductivities.

2.3. Fourier transformation

If frequency domain EM responses are available, the time-domain response can be obtained by carrying out an inverse Fourier transform. For ease of notation, we introduce the generic symbol H for an EM field component or its sensitivity in the frequency domain. Then, the transient response and its time derivative for a step turnoff of the current can be expressed by

$$h(t) = -\frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}[H(\omega)]}{\omega}\cos(\omega t)d\omega \quad (6)$$

and

$$\frac{\partial h(t)}{\partial t} = \frac{2}{\pi}\int_0^\infty \mathrm{Im}[H(\omega)]\sin(\omega t)d\omega, \quad (7)$$

where $\mathrm{Im}[H(\omega)]$ is the imaginary part of the EM field component or its sensitivity. Eqs. (6) and (7) correspond to the step and impulse responses, respectively. Numerical evaluation of these sine and cosine transforms can be performed with Anderson's (1975) digital filters (Newman et al., 1986).

2.4. Verification of forward solution

To verify the accuracy of our solution for TEM responses, we compared it with a 1-D semi-analytic solution. The model consists of an anomalous layer with a resistivity of 5 Ω-m and a thickness of 600 m buried 400 m deep in a half-space with a resistivity of 100 Ω-m. The source is a 2-km-long grounded wire on the surface. The model was discretized into an 81 × 59 × 36 mesh, and the TEM responses were computed from 30 frequency responses from 0.01 Hz to 1 MHz using Eqs. (6) and (7). Fig. 1 shows the vertical component of magnetic field h_z and its time derivative $\partial h_z/\partial t$ computed for a receiving point of 3 km from and on the perpendicular bisector of the wire. Our numerical solutions are in good agreement with the semi-analytic solutions.