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Optimized arrays for 2-D resistivity survey lines with a large number of electrodes

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ABSTRACT

Previous studies show that optimized arrays generated using the 'Compare R' method have significantly better resolution than conventional arrays. This method determines the optimum set of arrays by selecting those that give the maximum model resolution. The number of possible arrays (the comprehensive data set) increases with the fourth power of the number of electrodes. The optimization method faces practical limitations for 2-D survey lines with more than 60 electrodes where the number of possible arrays exceeds a million. Several techniques are proposed to reduce the calculation time for such survey lines. A single-precision version of the 'Compare R' algorithm using a new ranking function reduces the calculation time by two to eight times while providing results similar to the double-precision version. Recent improvements in computer GPU technology can reduce the calculation time by about seven times. The calculation time is reduced by half by using the fact that arrays that are symmetrical about the center of the line produce identical changes in the model resolution values. It is further reduced by more than thirty times by calculating the Sherman–Morrison update for all the possible two-electrode combinations, which are then used to calculate the model resolution values for the four-electrode arrays. The calculation time is reduced by more then ten times by using a subset of the comprehensive data set consisting of only symmetrical arrays. Tests with a synthetic model and field data set show that optimized arrays derived from this subset produce inversion models with differences of less than 10% from those derived using the full comprehensive data set. The optimized data sets produced models that are more accurate than the Wenner–Schlumberger array data sets in all the tests.

1. Introduction

In the last twenty-five years there have been major developments in the resistivity method where it can now provide realistic images in geologically complex areas. Developments in multi-electrode resistivity meter systems and rapid inversion software have led to the widespread use of two-dimensional (2-D) and even three-dimensional (3-D) resistivity surveys in engineering, environmental and mineral exploration surveys (Auken et al., 2006; Loke et al., 2013). 2-D surveys have proven to be the most economically efficient method in moderately complex areas, while 3-D surveys are usually reserved for more complex areas. Most of the early multi-electrode instruments have up to about 50 independent electrodes, but newer commercial instruments with more than 100 nodes are becoming more common.

There has been significant progress in automatic techniques to find the optimum set of electrode array configurations (Stummer et al., 2004; Maurer et al., 2010) in recent years. The 'Compare R' method by Wilkinson et al. (2006) proved to provide arrays with the highest resolution among the techniques examined by Loke et al. (2010a). The optimized arrays can significantly improve the resolution obtained with 2-D and 3-D surveys (Wilkinson et al., 2012; Loke et al., 2014c). Previous research have concentrated on improving the depth resolution for survey lines with a small number (30 to 50) of electrode positions. The 'Compare R' method faces practical computational limitations for longer survey lines (Blome et al., 2011). Thus the main focus of this paper is on techniques to reduce the calculation time required to generate the optimized arrays for lines with a large number of electrodes. Other practical aspects on the use of optimized arrays for field surveys (such as the reduction of electrode polarization noise and use of data error estimates) can be found in Wilkinson et al. (2012).

The following section gives a brief description of the 'Compare R' array optimization method, followed by computational and numerical techniques devised to reduce the calculation time required to generate

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the arrays. Results from a synthetic model and a field survey are then presented.

2. Theory

The smoothness-constrained least-squares optimization method is frequently used for 2-D inversion of resistivity data (Ellis and Oldenburg, 1994; Loke et al., 2003). The subsurface model commonly consists of a large number of rectangular cells where the size and positions of the cells are fixed but the resistivity is allowed to vary. The least-squares equation that gives the relationship between the model parameters and the measured data is given by

$$\left(\mathbf{G}^{\mathbf{T}}\mathbf{G}+\lambda\mathbf{C}\right)\Delta\mathbf{r_i}=\mathbf{G}^{\mathbf{T}}\mathbf{d}-\lambda\mathbf{C}\mathbf{r_{i-1}}. \quad (1)$$

The Jacobian matrix **G** contains the sensitivities of the (logarithm of) calculated apparent resistivity values with respect to the (logarithm of) the model resistivity values **r**. **C** contains the roughness filter constraint, λ is the damping factor and **d** is the data misfit vector. $\Delta\mathbf{r_i}$ is the change in the model parameters. Various modifications have been made to the above equation to incorporate desired characteristics in the data misfit or model roughness functions, such as a L1-norm criterion for the data misfit and model roughness filter (Farquharson and Oldenburg, 1998; Loke et al., 2003) and to include a data error weighting matrix (Ellis and Oldenburg, 1994). The model resolution matrix **R** (Menke, 1989; Loke et al., 2010a) is given by

$$\mathbf{R}=\mathbf{BA}, \text{where} \quad \mathbf{A}=\mathbf{G}^{\mathbf{T}}\mathbf{G} \text{ and } \mathbf{B}=\left(\mathbf{G}^{\mathbf{T}}\mathbf{G}+\lambda\mathbf{C}\right)^{-1}. \quad (2)$$

The main diagonal elements of **R** give an estimate of the model cells resolution. The 'Compare R' method (Wilkinson et al., 2006) attempts to determine the set of array configurations that will maximize the average resolution value. There are $E(E-1)(E-2)(E-3)/8$ independent four-electrode configurations for a system with E electrodes. Arrays where the current and potential electrodes are interleaved of the 'gamma' (Carpenter and Habberjam, 1956) type as well as those with large geometric factors are excluded to reduce the number of possible configurations (Stummer et al., 2004). The remaining configurations form the 'comprehensive' data set. A small base data set consisting of the dipole–dipole configurations with the 'a' dipole length of 1 unit electrode spacing (and maximum 'n' value of 6 to 10 depending on the maximum geometric factor set) is used as the starting base data set. The change in the model resolution for each new array when added to the base set is then calculated. A selected number of the configurations that gives the largest increase in the model resolution, and has a suitable degree of orthogonality to the existing configurations, is then added to the base data set (Wilkinson et al., 2012). The model resolution values for the new base data set are then recalculated using Eq. (2). This is repeated until the desired number of optimized arrays is selected.

The Sherman–Morrison Rank-1 update is used to calculate the change in the model resolution matrix $\Delta\mathbf{R_b}$ when a new configuration is added to the base set using the following equation (Loke et al., 2010b).

$$\Delta\mathbf{R_b}=\frac{\mathbf{z}}{1+\mu}\left(\mathbf{g}^{\mathbf{T}}-\mathbf{y}^{\mathbf{T}}\right)$$
$$\mathbf{z}=\mathbf{B_b}\mathbf{g}, \mathbf{y}=\mathbf{A_b}\mathbf{z} \text{ and } \mu=\mathbf{g}.\mathbf{z} \quad (3)$$

$\mathbf{A_b}$ and $\mathbf{B_b}$ are the matrices in Eq. (2) for the base data set. The vector **g** contains the model sensitivity values for the new array. The following function F_{CR} (Wilkinson et al., 2012) is used to the rank the improvement in the resolution for a model with m cells due to an add-on array.

$$F_{CR}=\frac{1}{m}\sum_{j=1}^{j=m}\frac{\Delta R_b(j,j)}{R_c(j,j)}. \quad (4)$$

R_c is the model resolution for the comprehensive data set. The average relative model resolution given by

$$S_r=\frac{1}{m}\sum_{j=1}^{j=m}\frac{R_b(j,j)}{R_c(j,j)}, \quad (5)$$

is commonly used to assess the performance of the optimized arrays. In the following sections, a value of 0.001 (Wilkinson et al., 2012) is used for the damping factor λ in Eq. (2) for the model resolution calculations. We also use the simple form of the model resolution equation where the identity matrix **I** is used in place of **C** (Wilkinson et al., 2006). However, the proposed techniques can be also used for different variations of this equation, such as using a roughness filter matrix (Loke et al., 2010b) or incorporating a data weighting matrix (Wilkinson et al., 2012).

3. Methods to reduce the calculation time

A number of numerical and computational techniques that were developed to reduce the calculation time are described in the section.

3.1. Using the symmetry of 2-D survey lines and models

A modification was introduced by Loke et al. (2010a) whereby whenever an array was added to the optimized data set, a check was made to ensure the symmetrical counterpart of the array was also included. This ensures that over a symmetrical structure the optimized data set will also display a symmetrical anomaly to avoid any bias in the data. Fig. 1 shows an example of a symmetrical pair of arrays for a survey line with 17 electrodes. Assuming the initial base data set and model discretization of the subsurface into rectangular cells are symmetrical about the center of the line, both arrays will give the same value for the model resolution change function F_{CR} when added to the base set. While the array in Fig. 1a will cause a larger change in the model resolution (ΔR_b) values for the model cells under the left half of the line, and the array in Fig. 1b will have a larger effect on the right half, the sum of the relative changes for all the cells will be the same. Thus it is only necessary to calculate F_{CR} for one of the arrays. The time taken by the 'Compare R' method is proportional to the number of arrays in the comprehensive data set. As an example, if a maximum geometric factor of 4146.9 m is used (corresponding to a dipole–dipole array with $a=1$ m and $n=10$), the comprehensive data set for a survey line with 80 electrodes has 2,973,047 arrays but slightly less than half (1,485,564 or 49.97%) are the symmetrical counterparts. The calculation time is reduced by about half by using the symmetry in the model resolution values. A small number of arrays that are symmetrical about the center of the line of electrodes (Fig. 1c) do not have a symmetrical counterpart in the comprehensive data set. We note that 3-D surveys using a rectangular grid of electrodes have a four-way symmetry (Loke et al., 2014c), so a similar technique can be used to reduce the calculation time for 3-D optimized arrays by nearly three-quarters.

For models with non-symmetrical topography or subsurface resistivity variations the assumption of symmetrical variations in the model resolution values is no longer valid. However, from previous field surveys, it has been found that optimized arrays calculated based on a simple homogeneous model with a flat surface still performed much better than conventional arrays in areas with significant topography and large resistivity variations (Stummer et al., 2004; Wilkinson et al., 2012).