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Full waveform inversion method using envelope objective function without low frequency data

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A R T I C L E I N F O

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A B S T R A C T

Full waveform inversion (FWI) has been a successful tool to build high resolution velocity models, but it is affected by a local minima problem. The conventional multi-scale strategy to tackle this severe problem may not work for real seismic data without long offsets and low frequency data. We use an envelope-based objective function FWI method to provide the long wavelength components of the velocity model for the traditional FWI. The gradient can be computed efficiently with the adjoint state method without any additional computational cost. Simple models are used to prove that the envelope-based objective function is more convex than the traditional misfit function, thus the cycle-skipping problem can be mitigated. Due to the envelope demodulation effect, the adjoint source of the envelope-based FWI contains abundant low frequency information, therefore the gradient tends to sense the low wavenumber model update. A Marmousi synthetic data example illustrates that the envelope-based FWI method can provide an adequately accurate initial model for the traditional FWI approach even when the initial model is far from the true model and low-frequency data are missing.

1. Introduction

Quantitative seismic imaging of subsurface parameters is still one of the main challenges for oil and gas reservoir characterization. Full waveform inversion (FWI) allows us to achieve high-resolution quantitative models of the subsurface by exploiting the full information of pre-stack seismic data (Tarantola, 1984). In practice, this method is an ill-posed and highly nonlinear inverse problem, which is sensitive to noise, inaccuracies in the starting model, and a lack of low frequencies in seismic data (Wang and Rao, 2006; Virieux and Operto, 2009). With the adjoint state method, the first gradient of FWI is recognized as migration (Tarantola, 1984), which can only provide the short-wavelength components of the velocity model. However, when low-frequency data are missing and the acquisition aperture is narrow, major difficulties are encountered updating the long-wavelength components of the model.

The low frequency information is fairly important for traditional full waveform inversion and inversion tends to a local minimum if the starting model is not in the vicinity of the global minimum (Gauthier et al., 1986). This severe local minimum problem is caused by the oscillatory nature of seismic data where high frequency data behave in a non-convex way at the low wavenumbers of the velocity model (Mulder and Plessix, 2008). The low-frequency components can be used to avoid the local minima because the low frequency objective function has a smaller number of local minima than the high frequency objective function (Bunks et al., 1995). Baeten et al. (2013) illustrated the benefits of low frequencies in acoustic full waveform inversions and impedance estimates. Without the use of first scale low frequency data, FWI fails to retrieve a reasonable background velocity. Because the width of the basin of the global minimum for low frequency data is wide, to avoid the local minima, successful FWI cases require very low frequency information in real seismic data and/or an initial model of adequate accuracy, which causes the modeled data to be, at most, a half cycle away from the real model (Alkhalifah and Choi, 2012).

To mitigate the local minimum problem, a series of inversion strategies have been proposed. In the time domain, a multi-scale strategy is presented to mitigate the nonlinearity of the inverse problem by successively inverting band-passed data from lower to higher frequency (Bunks et al., 1995). Low frequency data are not easily affected by the cycle-skipping problem, therefore nonlinearity can be mitigated to some extent using this strategy. The frequency domain provides a more natural multi-resolution framework for this multi-scale approach by performing successive inversions of increasing frequencies to mitigate the nonlinearity of the inverse problem associated with the high-frequency cycle-skipping artifacts (Sirgue and Pratt, 2004). Another advantage of the frequency domain inversion is that it can provide an accurate result with only a limited number of frequencies using the optimized strategy (Sirgue and Pratt, 2004). The multi-scale approach might not be sufficient to provide a reliable FWI result for realistic frequencies and starting models in the case of complex structures. Another effective strategy is to select some subsets of seismic arrivals. The aim of the time windowing strategy is to perform a heuristic selection of aperture angles in the data to introduce some regularization effects and also to mitigate the strong

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nonlinearity. Using a narrow time window centering on the first arrival leads to early-arrival waveform tomography (Sheng et al., 2006). The layer stripping inversion strategy that proceeds hierarchically from the shallow to the deep part in FWI can be applied using combined offset and temporal windowing (Shipp and Singh, 2002; Wang and Rao, 2009). In the frequency domain, Brossier et al. (2009) used complex valued frequencies to simplify the complex seismic data and a two-level hierarchy inversion strategy to further mitigate the nonlinearity of FWI. However, low frequencies and long offsets are not always present in the real data and the lack of low-frequency data (<4 Hz) in conventional seismic exploration data often hinders the time domain multi-scale method or the frequency continuation strategy.

As a result that we have to obtain the initial model for FWI without low frequency information, an alternative approach is to modify the objective function to match some secondary observables of seismic data, which can restore the ability of high frequency data to invert the smooth model. For example, Luo and Schuster (1991) used cross-correlation to measure the mismatch of the first-arrival travel-time and Zhang et al. (2011) improved this method by using reflection data to image the deep part of the model. Liu et al. (2009) developed the Fresnel volume tomography using the first-arrival traveltime obtained by picking or cross-correlation to invert the near-surface structure. Because cross-correlation is meaningful for waveforms with similar shapes, phases have to be isolated in seismic data. Cross-correlation objective functions can weaken the high nonlinearity of the inverse problem, but it needs manual picking or time windowing, which is time consuming. Laplace-domain waveform inversion (Shin and Cha, 2008) can provide a smooth model even when the starting model is greatly different from the exact one. Using a damping function to simplify complex seismic data and matching the weighted amplitudes of both first and later arrivals makes the Laplace-domain waveform inversion behave in a more linear manner to recover long-wavelength velocity structure (Bae et al., 2012). However, this method requires long offset data to image the deep part of the model and is sensitive to the noise before first arrivals. In the frequency domain, Shin and Min (2006) suggested using logarithmic wavefields to isolate the amplitude and phase contributions. However, phase approaches of waveform inversion are based on extracting the phase without eliminating the wrapping phenomena. Using the phase derivative to construct the objective function is not affected by the wrapping effect (Choi and Alkhalifah, 2013). Phase derivative with a strong damping or the instantaneous travel-time inversion can provide a good initial model for full waveform inversion and migration. Bozdağ et al. (2011) used instantaneous phase and envelope misfits to construct an objective function in full waveform inversion. Working with the Hilbert transform can avoid non-linear mixtures of phase and amplitude (Bozdağ et al., 2011). In phase measurements, the serious phase-wrapping problem can be addressed by using long period waveforms first, which may not be captured in seismic exploration data. Although Bozdağ et al. (2011) made qualitative comparisons of the corresponding finite-frequency sensitivity kernels (Wang, 2011) to gain insight on the advantages and disadvantages of their chosen objective functions, they did not show any inversion examples using instantaneous phase or envelope measurements. They also did not give a detailed explanation why the envelope-based objective function is better than the traditional waveform misfit.

In this paper, we develop the idea of Bozdağ et al. and use an envelope to construct the objective function of the full waveform inversion in data domain. Our aim is to extract as much information as possible from a single seismogram and to present inversion strategies to mitigate the well-known nonlinearity problem of full waveform inversion, in particular, when low frequency data are missing. To demonstrate that the envelope-based objective function performs better than the traditional waveform residual objective function, we focus our analysis specifically on objective functions, adjoint sources and the non-linearity issue. To provide better insight into the advantages of our chosen misfit function, we further discuss the influence of missing low-frequency data on different objective functions, which is an important issue to make waveform inversion more practical. Numerical experiments are included to illustrate that the envelope-based FWI can update long wavelength models and provide a better initial model for the traditional FWI.

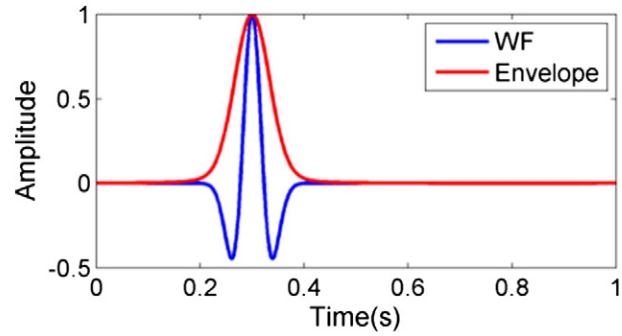

Fig. 1. Ricker wavelet and its envelope.

2. Theory

In waveform inversion, the difference between the modeled and observed wavefields is minimized to update the subsurface velocity model. Since objective functions, adjoint sources and the non-linearity issue play a central role in the success of the gradient method for full waveform inversion, we specifically focus our analysis on these three aspects to compare the differences between the envelope-based and waveform-based objective functions.

2.1. Waveform and envelope misfits

The FWI results rely entirely on the seismic information, which is used to construct the objective functions. The traditional full waveform inversion method estimates the subsurface model by minimizing the misfit between the calculated and observed data. The corresponding objective function can be defined as

$$J(\mathbf{m}) = \frac{1}{2}\sum_{s,r}\int_0^T F[u(\mathbf{x}_s,\mathbf{x}_r,t),u_0(\mathbf{x}_s,\mathbf{x}_r,t)]dt, \tag{1}$$

where $u(\mathbf{x}_s, \mathbf{x}_r, t)$ is the calculated data and $u_0(\mathbf{x}_s, \mathbf{x}_r, t)$ is observed data at given source $\mathbf{x}_s$ and receiver $\mathbf{x}_r$ locations. $F[u(\mathbf{x}_s, \mathbf{x}_r, t), u_0(\mathbf{x}_s, \mathbf{x}_r, t)]$ is a function of $u(\mathbf{x}_s, \mathbf{x}_r, t)$ and $u_0(\mathbf{x}_s, \mathbf{x}_r, t)$, which measures the matching degree between the calculated and observed data. We usually use a L_2 norm difference objective function

$$J(\mathbf{m}) = \frac{1}{2}\sum_{s,r}\int_0^T [u(\mathbf{x}_s,\mathbf{x}_r,t)-u_0(\mathbf{x}_s,\mathbf{x}_r,t)]^2 dt. \tag{2}$$

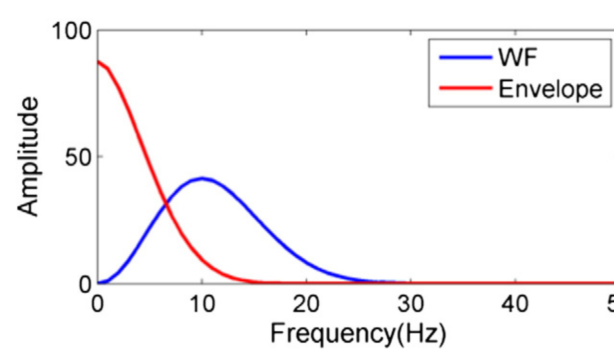

Fig. 2. Spectra of the Ricker wavelet and its envelope.