Journal of Applied Geophysics 109 (2014) 62–70

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Seismic data denoising based on the fractional Fourier transformation

Ming-Yue Zhai *

North China Electric Power University, Beijing, China

ARTICLE INFO

Article history:
Received 6 June 2014
Accepted 18 July 2014
Available online 1 August 2014

Keywords:
Seismic data
Noise mitigation
The fractional Fourier transform
Time–frequency

ABSTRACT

Seismic data may suffer from too severe noise contamination to carry out further processing and interpretation procedure. In the paper, a new scheme was proposed based on the fractional Fourier transform (FrFT) in time frequency domain to mitigate noise. The scheme consists of two steps. In the first step, the seismic signal is filtered with the ordinary Butterworth filter in the frequency domain. The residual noises after frequency filtering are with the same frequencies with the filtered seismic signals. In order to mitigate the residual noises further, the FrFT filter is applied in the second step. The results from the simulated seismic signals and the measurements data verify the validity of the proposed scheme in both frequency and time–frequency domains.

1. Introduction

Subsurface image construction and rock property estimation within the earth are essential tasks for seismic exploration. Many types of data, however, may suffer from too severe noise contamination to carry out such tasks. Random noise can adversely affect seismic data analysis and should be suppressed before data processing and interpretation. In many cases, therefore, noise mitigation is essential in order to extract useful information from the raw measurements (Beckouche and Ma, 2014; Bonar and Sacchi, 2012).

Fundamentally, noises/interferences mitigation is implemented in various domains, where noises/interferences can be separated clearly. Frequency domain is the first one to perform such tasks. Following it, various domains have been explored in seismic signal processing, such as $f \sim x$ domain (Chen and Ma, 2014), $\lambda \sim f$ domain (Forghani et al., 2012), and $f \sim k$ domains (Hennenfent and Herrmann, 2006; Hennenfent et al., 2010). As the generalization of such domains, Radon transform and edge-preserving smoothing and scalar median filter (SMF) are also commonly used (Ibrahim and Sacchi, 2014; Oropeza and Sacchi, 2011). One of the advantages of such extended technologies is that less smearing among adjacent samples is produced after noise attenuation compared to other methods. For example, the SMF can remove an abnormal impulse from seismic records without smearing the impulse into its nearby samples as the mean or $f \sim x$ filters (Hui-Qun and Zhi-Xian, 2014).

More recently, the curvelet domain has been presented as the methodology for adaptive subtraction (Kustowski et al., 2013; Liang et al., 2014). The curvelet transform decomposes data into a linear, weighted sum of curvelets, with each curvelet like a small piece of wavefront or reflector. Such curvelets are parameterized by a dominant period, a dominant dip/velocity, and two location coordinates.

Noise mitigation in curvelet domain has proven to be an excellent technique for suppression of incoherent, as well as coherent noise in seismic data. The increased level of parameterization relative to the well known $f \sim k$ domain, allows for a more refined characterization of the noise. It also reduces the likelihood of signal and noise overlap. Altering any curvelet coefficient has a local impact, with a smaller chance of damaging the signal of interest compared to the global transforms, such as the Fourier transform in frequency domain.

The basic implementation of curvelet noise mitigation involves thresholding of the coefficients in the curvelet domain and it can handle only data with a relatively constant level of incoherent noise (Liu, 2013).

Recent years also see the sparse representations of seismic data have played a more and more important role in seismic noise mitigation as the generalizations of the curvelet domain. New methods based on sparse representations include shearlets (Ma and Plonka, 2010), and contourlets (Neelamani et al., 2008). Therefore, sparse representations in a transform domain offer a promising framework to seismic-data denoising.

Another promising domain, named time–frequency, has also seen the applications in seismic exploration. For example, the optimal Gabor transform, based on the time–frequency rotation property of the fractional Fourier transform, is used to carry out the spectrum decomposition of the seismic signal. Results show that the instantaneous frequency slices obtained in time–frequency domain are superior to

* Tel.: +86 10 61773737.
E-mail address: mingyue.zhai@gmail.com.

http://dx.doi.org/10.1016/j.jappgeo.2014.07.012
0926-9851/

the ones obtained by the traditional domain (Chen et al., 2013; Zhang et al., 2010).

However, there is no concerning of application of time–frequency to noise/interference mitigation yet. In the paper, we propose a new scheme based on the fractional Fourier transform (FrFT) in time–frequency domain to mitigate noise. In the scheme, an FrFT-based signal decomposition algorithm is utilized to decompose received seismic signals into a linear combination of signal components in the FrFT domain. As a transformation tool, the Gabor representation is employed to estimate the maximum amplitude response in the fractional transform domain. Furthermore, seismic signals are decomposed into the signal components and noisy components, whose locations are separated clearly in the FrFT domain. Then, a reverse FrFT transform is applied to the separated signals to recover the expected signals, while the noises will be mitigated greatly. Analytical and simulation results show that the proposed noise mitigation based on the FrFT is an alternative method to perform high signal-noise-ratio (SNR) performance of seismic signals.

In the paper, we evaluate the performance of the noise mitigation based on the FrFT representations in seismic signal processing applications. The paper is outlined in the following way. The fundamentals of the FrFT can be found in Section 2, while the proposed method to mitigate the noises/inferences is detailed in Section 3. In Section 4, the performance of the proposed scheme is presented with applications to various simulated seismic signals and the measurement data. The paper is concluded in Section 5.

2. Fundamentals of fractional Fourier transformation

The idea of fractional powers of the Fourier transform operator appears in the mathematical literature as early as 1929.

Later on it was used in quantum mechanics and signal processing (Mcbride and Kerr, 1987; Namias, 1980; Zhi-Na et al., 2013).

The continuous fractional Fourier transform can be defined as:

$$f_a(\xi) = \int_{-\infty}^{+\infty} K_a(\xi, x) f(x) dx \tag{1}$$

where the kernel $K_a(\xi, x)$ is defined as follows. Set $\alpha = \frac{a\pi}{2}$ then

$$K_a(\xi, x) = \frac{e^{-\frac{i\pi\,\mathrm{sgn}(\sin\alpha)-2\alpha}{4}}}{\sqrt{|\sin\alpha|}} e^{-i\pi\left[2\frac{x\xi}{\sin\alpha}-\left(x^2+\xi^2\right)\cot\alpha\right]}. \tag{2}$$

Note that for $a \in \mathbb{Z}$, the FrFT is with some special properties that we can use. For example, if $a = 4k$, $k \in \mathbb{Z}$, the FrFT becomes the identity $f_{4k}(\xi, x) = f(\xi)$, hence the kernel is in that case

$$K_{4k}(\xi, x) = \delta(\xi - x), k \in \mathbb{Z} \tag{3}$$

and for $a \in 2 + 4\mathbb{Z}$, this is the parity operator $f_{2+4k}(\xi, x) = f(-\xi)$, corresponding to the kernel $f_{2+4k}(\xi, x) = \delta(\xi + x)$.

In addition, the following properties are also very important in signal separation.

For $a \in 1 + 4\mathbb{Z}$, $\mathcal{F}^a = \mathcal{F}^1 = \mathcal{F}$ is just the Fourier operator $\mathcal{F}$, and for $a \in 3 + 4\mathbb{Z}$, $\mathcal{F}^a = \mathcal{F}^3 = \mathcal{F}^2\,\mathcal{F}$ in other words, if $f_1(\xi) = \mathcal{F}[f(x)]$ is the Fourier transform, then $f_3(\xi) = f_1(-\xi)$.

This should make clear that $\mathcal{F}^a$ can be interpreted as the ath power of the Fourier transform which may be interpreted modulo 4. So, we have for example the well known properties $\mathcal{F}^a\mathcal{F}^b = \mathcal{F}^{a+b}$, and $\mathcal{F}^a\mathcal{F}^{-a} = I$ is the identity.

Like for the Fourier transform, there exists a discrete version of the fractional Fourier transform. It is based on an eigenvalue decomposition of the discrete Fourier transform matrix. A signal sequence can be written as

$$\begin{aligned}\mathbf{f} &= [f(x_0), f(x_0+\Delta), \mathcal{F}(x_0+2\Delta), \ldots, f(x_0+N\Delta-\Delta)]^T. \\ &= [f_0, f_1, \ldots, f_{N-1}]^T.\end{aligned} \tag{3}$$

The discrete Fourier transform of this vector is defined as the vector $\mathbf{f}_a = \mathcal{F}^a[\mathbf{f}]|_{a=1}$, and can be expressed as

$$\mathbf{f}_a|_{a=1} = \mathbf{Ff} \tag{4}$$

where the $N \times N$ DFT matrix $\mathbf{F}$ has entries that are the Nth roots of unity: $\mathbf{F}(k, n) = \frac{1}{\sqrt{N}} W^{kn}$ with $W = e^{-i\frac{2\pi}{N}}$.

Hence

$$\mathbf{f_1}(k) = \sum_{n=0}^{N-1} \mathbf{F}(k, n) f(n) = \sum_{n=0}^{N-1} f(n) e^{-i\frac{2\pi k}{N}} \tag{5}$$

If the discrete Fourier transform matrix $F = E\Lambda E^{-1}$ is the decomposition we mentioned before, then $F = E\Lambda^a E^{-1}$ is the corresponding discrete fractional Fourier transform.

It should be noted that the DFT matrix F satisfies $F^4 = I$ with I the identity matrix.

It has the eigenvalues $[1, -i, -1, i] = [e^{ik\pi/2}, k = 0, 1, \ldots, N-1]$.

It has also N independent orthonormal eigenvectors that can be arranged as the columns of a matrix E, so that its eigenvalue decomposition is $F = E\Lambda E^T$.

The definition of the discrete fractional Fourier transform is then easily given as a multiplication with a (fractional) power of the Fourier matrix.

3. The proposed noise mitigation based on the FrFT transform

3.1. Fundamentals of signal separation based on the FrFT

From the definitions, we can see that the FrFT can decompose a signal in one dimension (t domain in our case) into two dimensions ($[a, \xi]$ domain). The first dimension a, called the fractional order, is the fractional domain, while the second dimension ξ can be interpreted as different meaning according to the signal's property and the value of the first domain a. For example, if the signal is $f(t)$ in time domain, ξ can be interpreted as frequency when $a \in 1 + 4\mathbb{Z}$ according to the FrFT properties. Therefore, the FrFT adds an extra degree of freedom in signal processing, and hence there is a possibility that signals and noise/interference can be separable in some of the domains $[a, \xi]$.

One of the applications of FrFT is to separate signals from noise/interference in the FrFT domain where the conventional Fourier transform may fail. As discussed, a time-domain signal can be represented in multiple FrFT domains (because the order a can have different values), there is a potential to separate signals from noise/interference in some of the FrFT domains. As for numerical example, we used two chirp signals mixed together in time domain:

$$y(t) = A_0\left[e^{-i2\pi(f_1t+a_1)t} + e^{-i2\pi(f_2t+a_2)t}\right] \tag{6}$$

where the two chirp signals are with the same amplitude A_0.

The first chirp signal is with linear frequency function with time as $f_1t + a_1$, while the second one with the linear frequency with time as $f_2t + a_2$. The parameters are as follows: $f_1 = 10$, $a_1 = 0$, $f_1 = 10$ and $a_2 = 25$. Such signal, and its power spectra and time–frequency representations, are all illustrated in Fig. 1a. It is observed that these