Journal of Applied Geophysics 109 (2014) 111–118

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Seismic inversion based on L1-norm misfit function and total variation regularization

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ARTICLE INFO

Article history:
Received 15 May 2014
Accepted 26 July 2014
Available online 8 August 2014

Keywords:
L1-norm misfit function
Total variation regularization
Iteratively re-weighted least squares
Seismic inversion
A priori information constrain

ABSTRACT

Objective: To solve the inverse problems when outliers exist in the seismic data and discontinuities such as layer boundaries need to be clearly delineated and merge the low frequency information to the inverted parameters.
Methods: L1-norm misfit function, total variation regularization, a priori information constraints, method of Lagrange multipliers, and iteratively re-weighted least squares.
Results and conclusions: Integrating the L1-norm misfit function, total variation regularization and a priori information constraints via the method of Lagrange multipliers, we create the objective function of seismic inversion to solve the inverse problems that outliers exist in the seismic data and discontinuities such as layer boundaries need to be clearly delineated. In addition, the priori information constraints ensure the inverted parameters have low frequency components.
Practice: The proposed inversion method is successfully tested on noisy synthetic seismic data with outliers and real seismic data.
Implications: If there are a small number of outliers in the seismic data, we need to do the seismic inversion in a way that minimizes their effect on the estimated parameters. However, the L2-norm misfit function is highly susceptible to even small numbers of inconsistent seismic observations. As an alternative to L2-norm, one can consider the solution that minimizes the L1-norm misfit function (L1MF) which will be more outlier-resistant, or robust, than the L2-norm solution. Of course, there are some alternative techniques to find the favorable regularization parameters. A set of good regularization parameters is the key of the seismic inversion process.

1. Introduction

Robustness is an important property in seismic inversion strategy. The potential disadvantages of computing L2-norm misfit function to solve seismic inverse problems have long been assessed. Many scholars discussed a variety of robust procedures. Claerbout and Muir (1973) give many examples to illustrate the advantages of L1-norm misfit function inversion. They point out that these advantages are based on the fact that solutions to L2 norm misfit function tend to overstate the influence of outliers which may arise from the procedural measurement error, or other reasons, in the seismic data. However, there are some disadvantages in the L1-norm optimization. The biggest one is that the inverse problem is nonlinear. In order to solve this problem, the most useful method is simplex-based method for linear programming extensions of Gaussian method (Barrowdale and Roberts, 1974; Bloomfield and Steiger, 1984). The other techniques for L1-norm minimization methods are based on the interior-point and iteratively re-weighted least square (Aster et al., 2005, 2013; Coleman and Li, 1992; Portnoy and Koenker, 1997; Scales et al., 1988; Watson, 2000). The iteratively re-weighted least square (IRLS) method is the simplest way to implement in L1-norm optimization. It is most attributed by Schlossmacher (1973), Beaton and Tukey (1974). Byrd and Pyne (1979) provided the convergence results through numerous references to the use of IRLS, and so did Bissantz et al. (2009). Huber (1996) reviewed the history of methods for finding the robust solutions and discussed a variety of robust procedures.

In addition, total variation regularization (TVR) is appropriate for the inverse problems when we expect to estimate the discontinuities which are desirable in geologic environments with abrupt changes in P-wave impedance, such as carbonate caves, salt bodies, or strong faults. Methods for TVR are discussed in the following references (Osher and Fedkiw, 2002; Rodriguez, 2013; Varela, Verdin and Sen, 2006; Vogel, 2002). Due to the band-limited nature of seismic data, we add a priori information constraints as regularization terms to ensure the inverted parameters contain low frequency information.

Integrating the L1-norm misfit function, total variation regularization and a priori information constraints via the method of Lagrange multipliers (Hansen, 1992), the objective function of seismic inversion is created. In order to find the solution of this inverse problem, we use

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http://dx.doi.org/10.1016/j.jappgeo.2014.07.024
0926-9851/

IRLS strategy. The proposed method is tested on noisy synthetic seismic data with some outliers. At last, we perform this method using real seismic data to further verify its feasibility and stability.

2. Objective function

L2-norm misfit function solutions are highly sensitive to even small number of outliers which are data points highly discordant with the other seismic data. If there are some outliers in the seismic data due to incorrect measurements or other reasons, it is necessary to do the seismic inversion in a way that can minimize their effects on the inverted parameters. As an alternative to L2-norm misfit function, we consider to solve the inverse problem via minimizing the L1-norm misfit function, which is as the following

$$\|\mathbf{e}\|_1 = \|\mathbf{G}(\mathbf{m}) - \mathbf{d}\|_1 \tag{1}$$

where $\mathbf{e}$ is the vector of the residual, $\mathbf{d}$ is the vector of the seismic data, $\mathbf{G}$ is the forward operator, and $\mathbf{m}$ is the vector of the earth model parameters. For seismic inversion, $\mathbf{m}$ is the vector of P-wave impedance. The L1-norm misfit function solution will be more robust than the L2-norm one, because Eq. (1) does not square each of the terms in the misfit measurement. In Claerbout and Muir's words, the median value is more robust than the mean one (Claerbout and Muir, 1973).

The L1-norm misfit function solution is the maximum likelihood estimator for noisy seismic data corresponding to an exponential distribution

$$f(x) = \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|x-\mu|/\sigma} \tag{2}$$

where, x is a random variable, μ is the expectation, and σ is the standard deviation.

Seismic data distributed as exponential distributions are unusual. However, it is worthwhile to solve an inverse solution based on L1-norm misfit function rather than L2-norm misfit function. Even if most of the seismic data noise is distributed as normal distribution, there are reasons to doubt the existence of outliers.

In addition, total variation regularization is appropriate for the inverse problem when we expect to estimate the layer boundaries or edges. In the one-dimensional case, the first order TVR function is

$$TVR_1(\mathbf{m}) = \|\mathbf{T}_1\mathbf{m}\|_1 \tag{3}$$

and the second order TVR function

$$TVR_2(\mathbf{m}) = \|\mathbf{T}_2\mathbf{m}\|_1. \tag{4}$$

In Eqs. (3) and (4),

$$\mathbf{T}_1 = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \\ & & & & -1 & 1 \end{bmatrix} \tag{5}$$

$$\mathbf{T}_2 = \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 & 1 \end{bmatrix} \tag{6}$$

we can see that $\mathbf{T}_1$ is the first order finite difference operation, and $\mathbf{T}_2$ is the second order finite difference operation. For higher dimensions, $\mathbf{T}_1$ and $\mathbf{T}_2$ are often implemented as finite difference approximations to the gradient and Laplacian operators, respectively.

Here we hope to consider all the solutions with $\|\mathbf{T}_i\mathbf{m}\| < \delta$ and select the one which can minimize Eq. (1). So the inverse problem has the following form

$$\begin{cases} \min\|\mathbf{G}(\mathbf{m})\text{-}\mathbf{d}\|_1 \\ \|\mathbf{T}_i\mathbf{m}\|_1 < \delta \end{cases} \tag{7}$$

where, i = 1, 2.

Using the Lagrange multiplier technique, Eq. (7) can be turned into an unconstrained optimization problem

$$\min\|\mathbf{G}(\mathbf{m}) - \mathbf{d}\|_1 + \alpha\|\mathbf{T}_i\mathbf{m}\|_1 \tag{8}$$

where α is the regularization parameter for TVR.

Due to the band-limited nature of seismic data, a priori information constraint can be added as a regularization term to ensure the inverted parameters containing low frequency components. The contents of the a priori model $\mathbf{m}$ come from well log data, geological horizons, or other sources. In seismic inversion processing, according to the definition of reflection coefficient

$$r(t) = \frac{m(t+1) - m(t)}{m(t+1) + m(t)} \tag{9}$$

when $r(t)$ is small it can be approximated as

$$r(t) \approx \frac{\Delta m(t)}{2m(t)} \approx \frac{\partial}{\partial t}\frac{\ln m(t)}{2} \tag{10}$$

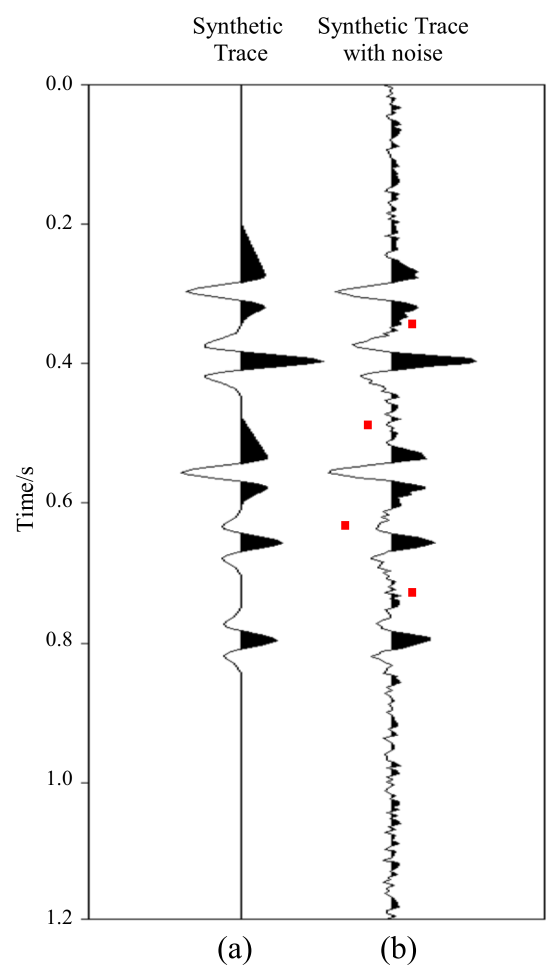

Fig. 1. (a) noise-free seismic data and (b) seismic data with 10% Gaussian random noise and outliers.