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A 3D staggered-grid finite difference scheme for poroelastic wave equation

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ABSTRACT

Three dimensional numerical modeling has been a viable tool for understanding wave propagation in real media. The poroelastic media can better describe the phenomena of hydrocarbon reservoirs than acoustic and elastic media. However, the numerical modeling in 3D poroelastic media demands significantly more computational capacity, including both computational time and memory. In this paper, we present a 3D poroelastic staggered-grid finite difference (SFD) scheme. During the procedure, parallel computing is implemented to reduce the computational time. Parallelization is based on domain decomposition, and communication between processors is performed using message passing interface (MPI). Parallel analysis shows that the parallelized SFD scheme significantly improves the simulation efficiency and 3D decomposition in domain is the most efficient. We also analyze the numerical dispersion and stability condition of the 3D poroelastic SFD method. Numerical results show that the 3D numerical simulation can provide a real description of wave propagation.

1. Introduction

Seismic modeling has been largely limited to the media of a single phase elastic solid. However, the environment of hydrocarbon reservoirs is a composite multiphase medium with gas and/or liquid occupying the voids between solid grains, which can be described by the theory of poroelasticity. The Biot (1956a,b, 1962) is the basis for the numerical simulation of wave propagation in fluid saturated poroelastic media.

A variety of different numerical methods have been used for poroelastic modeling, such as spectral method (Degrande and De Roeck, 1992), finite difference method (Gao and Zhang, 2013; Zhang and Gao, 2013), discontinuous Galerkin method (Dupuy et al., 2011), and finite volume method (Lemoine et al., 2013), etc. Due to the low memory requirement and computational cost, finite difference methods have been widely applied for porous wave equations.

Studies of wave propagation based on 3D finite difference have contributed to a better understanding of the source process, wave path effects, and basin structure response (Pitarka, 1999; Moczo et al., 2000). In order to handle the tremendous computational and memory requirements that are inherent to the 3D SFD technique, a variety of computational approaches have been developed and implemented. The most efficient parallel implementations are ones which are optimized to balance the computational load and minimize the communication between processors. This can be easily achieved for an explicit SFD method by domain decomposition (Bohlen, 2002; Sheen et al., 2006). Each processor solves the problem within its small subdomain and, at each time step, communication with neighboring processors using message passing interface (MPI) is performed to exchange wave field near the boundaries (Minkoff, 2002). In our study, we provide a 3D parallel staggered-grid finite difference method for poroelastic media using domain decomposition and MPI, we also provide the discussion on numerical dispersion and stability condition.

This paper is organized as follows. In Section 2, based on Biot's theory, we derive the 3D poroelastic wave equation in a first-order, velocity-stress system. Then, we present the staggered-grid finite difference scheme and give the dispersion relation for 3D poroelastic media; we also provide the stability condition in Section 3. In Section 4, a parallel finite difference scheme is implemented with domain decomposition and MPI, and the parallelization analysis is also presented. We give the numerical results of a homogeneous and a three layered poroelastic media in Section 5. Finally, we provide the conclusion of this paper in Section 6.

2. Wave equations for 3D poroelastic media

Biot (1956a) established the dynamic equations in a porous elastic solid saturated by a compressible viscous fluid. Dai et al. (1995)

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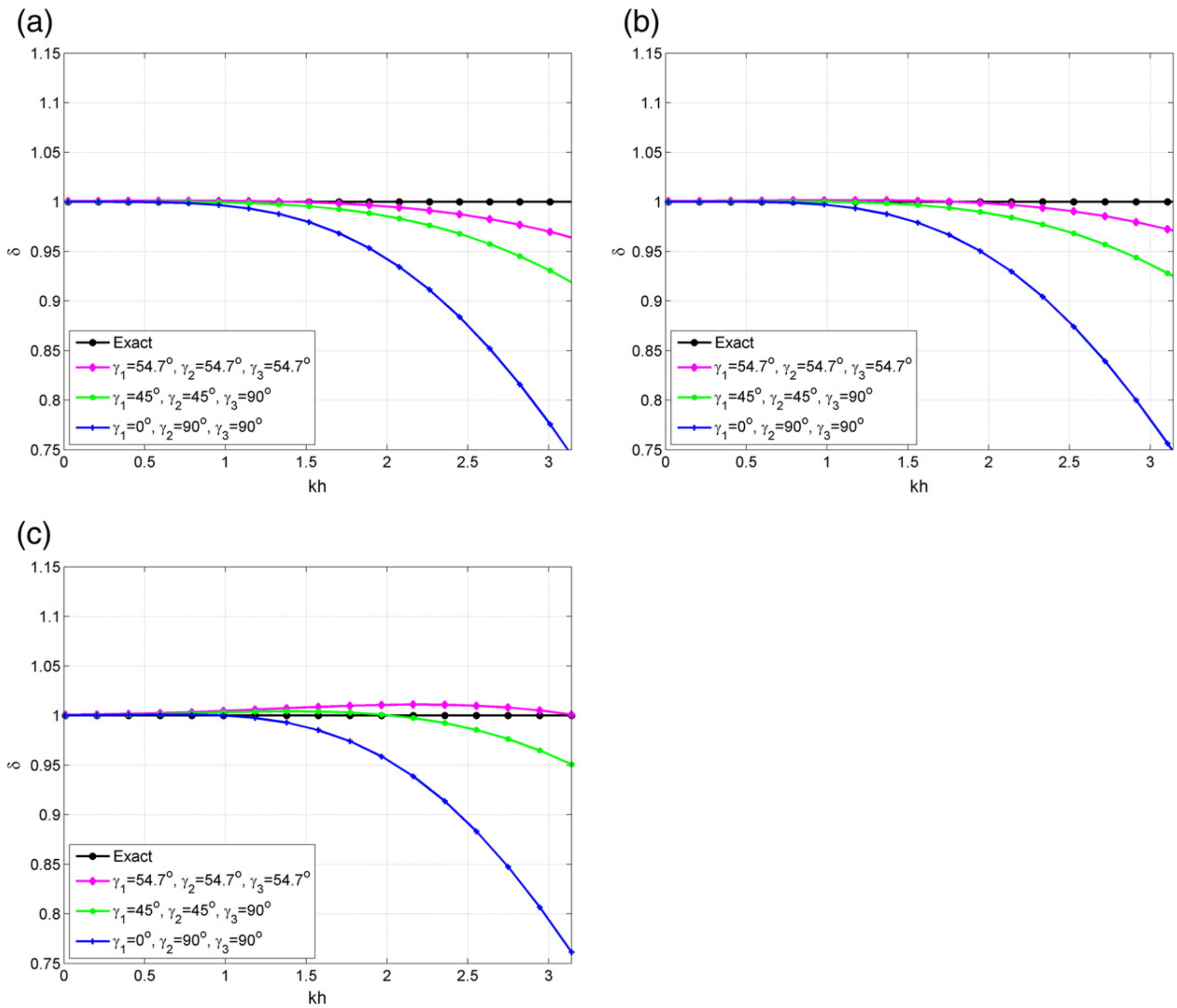

Fig. 1. Dispersion curves of (a) slow P wave, (b) S wave, (c) fast P wave at different propagation angles of a plane wave. $\tau = 0.5$ ms, $h = 5$ m, $M = 2$, $v_{sP} = 1180$ m/s, $v_S = 1790$ m/s, $v_{fP} = 3210$ m/s.

developed a first-order hyperbolic system that is equivalent to Biot's equations. Based on the wave equations in a 2D porous media mentioned above, we derive the velocity-stress formulation for a 3D poroelastic media.

According to Biot's theory, the equations of motion for a 3D porous media are given by

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{A}\frac{\partial \mathbf{u}}{\partial x} + \mathbf{B}\frac{\partial \mathbf{u}}{\partial y} + \mathbf{C}\frac{\partial \mathbf{u}}{\partial z} + \mathbf{D}\mathbf{u}, \tag{1}$$

here, the vector of unknowns is

$$\mathbf{u} = \left[v_x, v_y, v_z, w_x, w_y, w_z, \tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz}, s\right]^T, \tag{2}$$

where (v_x, v_y, v_z) is the solid particle velocity vector, (w_x, w_y, w_z) is the fluid particle velocity vector, and $(\tau_{xx}, \tau_{xy}, \tau_{xz}, \tau_{yy}, \tau_{yz}, \tau_{zz})$ is the solid stress tensor. The parameter s is related to the fluid pressure p and the porosity ϕ as follows,

$$s = -\phi p. \tag{3}$$

The matrixes of coefficients in Eq. (1) are presented in Appendix.

3. Dispersion analysis and stability condition

3.1. Staggered-grid finite difference scheme

In a staggered-grid scheme (Graves, 1996), the particle velocities, solid stress and fluid pressure are defined on different grid points, which can be represented by $(v_x)^n_{i,j,k}$, $(w_x)^n_{i,j,k}$, $(v_y)^n_{i+\frac{1}{2},j+\frac{1}{2},k}$, $(w_y)^n_{i+\frac{1}{2},j+\frac{1}{2},k}$, $(v_z)^n_{i+\frac{1}{2},j,k+\frac{1}{2}}$, $(w_z)^n_{i+\frac{1}{2},j,k+\frac{1}{2}}$, $(\tau_{xx})^{n-\frac{1}{2}}_{i,j,k+\frac{1}{2}}$, $(\tau_{yy})^{n-\frac{1}{2}}_{i,j,k+\frac{1}{2}}$, $(\tau_{zz})^{n-\frac{1}{2}}_{i,j,k+\frac{1}{2}}$, $(s)^{n-\frac{1}{2}}_{i,j,k+\frac{1}{2}}$, $(\tau_{xz})^{n-\frac{1}{2}}_{i+\frac{1}{2},j,k}$, $(\tau_{xy})^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k}$ and $(\tau_{yz})^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}$. The subscripts refer to the spatial indices, the superscripts are the time indices. For example, with a grid spacing of h and a time step of τ, the expression $(v_x)^n_{i,j,k}$ represents the x component of the solid particle velocity evaluated at point $[x = ih, y = jh, z = kh]$ and time $t = n\tau$.

The 2Mth - order staggered-grid finite difference scheme (Dong et al., 2000) for the first-order derivative with respect to space variable x can be written as

$$\frac{\partial f}{\partial x} = \frac{1}{h}\sum_{m=1}^{M} a_m \left\{ f\left[x + \frac{h}{2}(2m-1)\right] - f\left[x - \frac{h}{2}(2m-1)\right]\right\} + O\left(h^{2M}\right), \tag{4}$$

where f may be particle velocity, solid stress or fluid pressure; a_m ($m = 1, 2, \ldots, M$) are the staggered-grid finite difference coefficients.