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Rayleigh wave modeling: A study of dispersion curve sensitivity and methodology for calculating an initial model to be included in an inversion algorithm

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A B S T R A C T

This paper presents a study on Rayleigh wave modeling. After model implementation using Matlab software, unpublished studies were conducted of dispersion curve sensitivity to percentage changes in parameter values, including S- and P-wave velocities, substrate density, and layer thickness. The study of the sensitivity of dispersion curves demonstrated that parameters such as S-wave velocity and layer thickness cannot be ignored as inversion parameters, while P-wave velocity and density can be considered as known parameters since their influence is minimal. However, the results showed limitations that should be considered and overcome when choosing the known and unknown parameters through determining a good initial model or/and by gathering a priori information. A methodology considering the sensitivity study of dispersion curves was developed and evaluated to generate initial values (initial model) to be included in the local search inversion algorithm, clearly establishing initial favorable conditions for data inversion.

1. Introduction

The term seismic brings to mind the seismic methods of refraction and reflection, which are the most popular of those employed in geophysics. However, other seismic methods have been developed in recent decades, such as the seismic analysis of surface waves examined in this paper. In the early 1980s, Nazarian and Stokoe (1984) developed a new seismic method called the spectral analysis of surface waves (SASW), based on the study of Rayleigh waves (surface waves) normally present as noise in seismic recordings made during refraction and reflection surveys. A decade and a half later, Park et al. (1999) proposed a new seismic method based on the same physical principles as SASW but that sought to eliminate or reduce the deficiencies of the previous method, namely, to improve the signal-to-noise ratio, increase the efficiency of field surveys, and improve data processing. This method was called the multichannel analysis of surface waves (MASW) and is distinguished by the simultaneous use of 12 or more receivers in a linear array to ensure greater efficiency and flexibility of data acquisition, as well as a higher signal-to-noise ratio. MASW acquisition is similar to a refraction seismic survey but is distinguished by low-frequency (4.5-Hz) receivers in its linear array. The data acquired are presented as an offset–time (x–t) seismogram, where the data processing step begins. To separate Rayleigh waves from other waves, a two-dimensional Fourier transform is performed to obtain a frequency–wave number (f–k) amplitude spectrum (Horike, 1985) that can be mathematically transformed into a graph of the phase velocity as a function of the frequency, called the dispersion curve.

Because Rayleigh waves characteristically have greater energy than other waves, their greater amplitude enables them to be readily identified on an f–k amplitude spectrum. In vertically heterogeneous media, Rayleigh waves exhibit dispersive behavior, wherein the different wavelengths of the wave components penetrate to different depths, acquiring different phase velocities depending on variations in the mechanical properties of the medium. In other words, the velocity of a surface wave undergoing dispersion is not unique but, rather, is characterized by different values that are dependent upon the frequencies of the wave components. In addition, this dispersive characteristic caused by the heterogeneity of the medium is visible in the dispersion curve as what we refer to as its modal nature, which is a succession of curves wherein the first one that appears as the frequency increases, called the fundamental mode, is the curve normally used for modeling and data inversion. Because Rayleigh waves have this dispersion characteristic, they are not just a noise feature present in most seismic methods but can serve as a source of information in methods such as MASW.

Information on the substrate layers are contained in the dispersion curve, which, through modeling (the direct problem), allows the inversion (the inverse problem) of field data possible. Inversion entails solving a nonlinear problem using, for example, an iterative least-squares procedure as the Levenberg–Marquardt algorithm (Levenberg, 1944; Marquardt, 1963), which produces a model with layers of different thicknesses along with their respective shear velocities.

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Rayleigh surface wave methods enable the exploration of depths on the order of 100 m and have been successfully used in engineering/construction because, in contrast to refraction and reflection seismic methods, they allow inversion as a function of the S-wave velocity, which, in combination with substrate density, enables calculation of the substrate coefficient of rigidity. These methods, furthermore, can complement a refraction survey because the seismogram obtained by the latter method can be used to process and invert surface wave data.

1.1. Rayleigh wave modeling

Modeling Rayleigh waves involves the generation of dispersion curves (Fig. 1) from a model of heterogeneous layers with known physical parameters, such as density (ρ), P-wave velocity (α), S-wave velocity (β), and layer thickness (d). The modeling basically involves finding the roots of a nonlinear secular function

$$F(c, \omega, \rho, \alpha, \beta, d) = 0 \tag{1}$$

obtained from the equations derived from the theory of Rayleigh waves in a heterogeneous medium. Implementation of the theory used most for Rayleigh wave modeling is based on the method described by Dunkin (1965), a method modified from earlier approaches by Thomson (1950) and Haskell (1953).

To finding the roots of the secular function (Eq. (1)), the values of the parameters ρ, α, β, and d must be known and included in the equation as constants. In this way, for each frequency value (ω) included in the equation, the output will be one or more phase velocity values (c) representing the fundamental mode and, when they exist, the other modes present in the dispersion curves (Fig. 1).

Based on the theory of waves in a heterogeneous elastic medium with the imposition of appropriate boundary conditions, the secular function generated by the Dunkin approach is a recursive function. That is, its form depends on the number of layers in the model, since each layer represents a matrix of values and the secular function represents the multiplication of all of these matrices:

$$F(c,\omega) = \underbrace{\begin{pmatrix} T_{1212} \\ T_{1213} \\ T_{1214} \\ T_{1223} \\ T_{1224} \\ T_{1234} \end{pmatrix}^{T}}_{n} \underbrace{\begin{pmatrix} G_{1212} & G_{1213} & G_{1214} & G_{1223} & G_{1224} & G_{1234} \\ G_{1312} & G_{1313} & G_{1314} & G_{1323} & G_{1324} & G_{1334} \\ G_{1412} & G_{1413} & G_{1414} & G_{1423} & G_{1424} & G_{1434} \\ G_{2312} & G_{2313} & G_{2314} & G_{2323} & G_{2324} & G_{2334} \\ G_{2412} & G_{2413} & G_{2414} & G_{2423} & G_{2424} & G_{2434} \\ G_{3412} & G_{3413} & G_{3414} & G_{3423} & G_{3424} & G_{3434} \end{pmatrix}}_{n-1} \cdots \underbrace{\begin{pmatrix} G_{1212} \\ G_{1312} \\ G_{1412} \\ G_{2312} \\ G_{2412} \\ G_{3412} \end{pmatrix}}_{1}. \tag{2}$$

Layers

Each matrix of the equation contains the elements calculated for each layer thickness, beginning with the half-space and ending with the top layer. The T elements, for example, are the equations for the half-space and the G elements are the equations for the remaining layers.

The theoretical dispersion curve is obtained by finding the zeros of the secular function and is essential for subsequent data inversion. The difference between the real and theoretical dispersion curves yields what we call the object function, the minimum of which represents a good fit between theoretical and real data. Therefore, we utilize this object function to perform data inversion, which, being an iterative procedure, requires a set of initial values to start the inversion algorithm.

The data inversion of surface waves can be performed by means of global and local search procedures. Global search procedures, well studied by Yamanaka and Ishida (1996), Wathelet et al. (2004), and Ryden and Park (2006), offer the advantage of low dependence of the initial model. On the other hand, higher computational costs are involved when a large number of parameters are unknown. Local search procedures, in turn, require a good initial model to obtain appropriate data convergence. Therefore, once adopted, a good initial model becomes a great inversion procedure, accurate and fast.

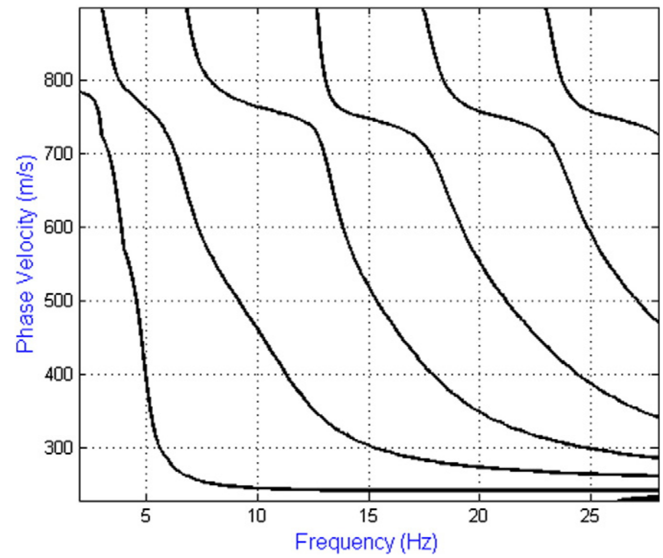

Fig. 1. Dispersion curves (modal nature).

For local methods, such as the Levenberg–Marquardt (Levenberg, 1944; Marquardt, 1963), the better the initial model, the better and faster it will converge to the final model that will be linked to the real geological substrate. According to Abbiss (1981), the maximum penetration depth of a shear wave is estimated to be between one-third and one-half of its wavelength; that is, the greatest wavelength (v/f) in a dispersion curve gives an estimate of the maximum investigation depth, with this estimate requiring conversion from phase velocity to S-wave velocity. The layer thickness parameter thereby becomes known and the S-wave velocities will be the parameter to be inverted, starting out from a set of initial values. Those values can be defined using the estimate provided by Abbiss for the maximum investigation depth

$$Z = P\lambda \tag{3}$$

and the values from the dispersion curve, where the value of parameter P lies between approximately 0.33 and 0.5.

2. Methodology

2.1. Rayleigh wave modeling

To be studied, the Rayleigh wave models needed to be implemented in Matlab programming language. Dunkin's (1965) algorithm was employed to generate the secular function. The fundamental mode of the dispersion curve was found by determining the first zero of the secular function for each frequency chosen. To achieve this, the numerical bisection procedure (Press et al., 1992) was implemented.

2.2. Dispersion curve sensitivity study

Dispersion curve sensitivity can be studied by calculating partial derivatives (Takeuchi et al., 1964; Xia et al., 1999) to check which parameter variations influence the dispersion curves more. However, the methodology implemented in this article enables further examination of the different trends of sensitivity to the variation of a single physical parameter. In addition, the graphical presentation of the results better illustrates the variations.

To study the sensitivity of the fundamental mode of dispersion curves, parameter sets including S-wave velocity (β_n), P-wave velocity (α_n), density (ρ_n), and layer thickness (d_n) were modified individually and jointly. Each parameter was varied by several percentages, allowing effects on dispersion curves to be checked.