Journal of Applied Geophysics 108 (2014) 152–158

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Determining the optimal order of fractional Gabor transform based on kurtosis maximization and its application

Lin Tian [a,b,*], Zhenming Peng [a]

[a] School of Optoelectronic Information, University of Electronic Science and Technology of China, China
[b] Department of Electronics and Information Engineering, Yili Normal University, China

ARTICLE INFO

Article history:
Received 16 November 2013
Accepted 24 June 2014
Available online 30 June 2014

Keywords:
Fractional Gabor transform
Kurtosis maximization
Optimal order searching
Spectral decomposition
Seismic signal

ABSTRACT

The optimal order of fractional Gabor transform (FrGT) has a direct effect on the results of fractional time–frequency analysis. This paper uses kurtosis maximization (KM) method to determine optimal order of FrGT, which is more suitable for non-stationary signals, and the corresponding algorithm fractional Gabor transform based on kurtosis maximization (KMFrGT) is proposed. Comparing with the amplitude maximization (AM), KM gives more rational choice in determining the optimal order of FrGT. The distributions with large kurtosis have sharper peaks than distributions have small one, which helps KMFrGT to give globally optimal time–frequency representations; AM method for optimal order of FrGT may lead time–frequency results to a local optimum, multi-chirp and quadratic chirp signals are used to demonstrate the effectiveness. The KMFrGT can suppress the Gaussian noise for the advantages of kurtosis; single seismic record added Gaussian noise is used to test the idea. The resolution of KMFrGT is higher than the traditional Gabor transform, a simple method is proposed to compare the resolutions of two transforms. The real field seismic data application results show the good performance of KMFrGT in time–frequency analysis.

1. Introduction

Spectral decomposition is introduced as a seismic interpretation technique by Partyka et al. (1999). Time–frequency analysis (TFA) has become an important practical seismic interpretation tool which has achieved widespread use. TFA is the key technology of seismic spectral decomposition; time–frequency resolutions determine whether the results of TFA are reliable. Among all the algorithms of improving the time–frequency resolutions, the methods of fractional Fourier transform (FrFT) combining traditional time–frequency transforms are excellent approaches, these transforms have some important properties: (a) Better resolutions can be achieved, for FrFT can be interpreted as a rotation of the signal in the time–frequency plane; (b) The computational complexity of FrFT is less expensive, for FrFT has fast algorithm (Ozaktas et al., 1996). As part of the research, fractional Gabor transform (FrGT) is derived from combining Gabor transform and FrFT (Akan et al., 2001; Zhang et al., 1997). FrGT has not been studied in depth; the main reason is that the transform results of FrGT are in fractional-time and fractional-frequency plane, the transform results have no clear physical meaning. To obtain time–frequency representation having clear physical meaning, mapping is needed from fractional-time and fractional-frequency plane to time–frequency plane (Capus and Brown, 2003). Durak and Arikan (2003) proposed fractional optimal short-time Fourier transform, which has high resolutions and the time-frequency representations have clear physical meaning. But this optimal method is feasible only for mono-component signals. Chen et al. (2013) utilized rotation properties of FrFT and gave optimal FrGT with novel window function. The optimal FrGT has clear physical meaning, i.e., the signal gets high resolutions time–frequency distribution (TFD) in fractional-time and fractional-frequency domain, and then rotates to the time–frequency plane.

The optimal order of FrGT is a crucial issue having to be confronted with, for it would directly affect time–frequency results of FrGT. There are two main methods available of searching the optimal order of FrGT: (1) the order having maximal amplitude in fractional domain is corresponding optimal order of FrGT (Chen et al., 2013). Fast algorithm of this method has been constructed (Zheng and Shi, 2010), this method is referred as amplitude maximization method for brevity; (2) the order having minimal time and frequency bandwidth is corresponding optimal order of FrGT (Durak and Arikan, 2003). Chen et al. (2013) investigated the equivalence of the two methods mentioned above using a linear chirp signal, to reduce computational cost, amplitude maximization method was adopted in the literature. Amplitude maximization is suitable for chirp signals, however, seismic signals are characterized as complex non-stationary signals, not simply chirp signals, and so new method is required for searching the optimal order of FrGT, which can be more suitable for seismic signals spectral decomposition.

* Corresponding author at: No. 4, Section 2, North Jianshe Road, Chengdu, 610054 Sichuan Province, China.
E-mail addresses: tianlin20110501@163.com (L. Tian), zmpeng@uestc.edu.cn (Z. Peng).

http://dx.doi.org/10.1016/j.jappgeo.2014.06.009
0926-9851/© 2014 Elsevier B.V. All rights reserved.

The advantages of kurtosis are as follows: kurtosis measures deviation from Gaussian distribution, the distribution has sharp peaks when the distribution has large kurtosis; conversely the distribution is flat; the kurtosis of Gaussian signals is zero which helps suppressing Gaussian noise. Kurtosis is a fourth-order statistic and higher-order statistics retain phase information, which can give the position information of frequencies distribution. For these advantages of kurtosis, in this paper, we will use kurtosis maximization seeking the optimal order of FrGT. For complicated non-stationary signals, amplitude maximization (AM) method may lead to failure; kurtosis maximization (KM) method can make rational choice for the optimal order. Optimal computational problems of FrGT can be solved.

2. Method

2.1. Fractional Gabor transform

N × N discrete FrFT transform matrix can be expressed as (Candan et al., 2000)

$$F_p = \sum_{k=0}^{N-1} u_k[n] e^{-\frac{i\pi}{2}kp} u_k[n], \quad 0 \le k, n \le N-1. \tag{1}$$

where u_k is the k-th order discrete Hermite–Gaussian function, F_p denotes the p-th order FrFT kernel matrix, i is the imaginary unit, p is the transform order of FrFT and the period of p is 4. F_p degenerates to Fourier transform kernel matrix with $p = 1$. FrFT is the representation of the signal in the fractional domain.

Consider an N dimensions signal in time domain $x(n)$ and its p-th order FrFT is

$$X_p(u) = \sum_{n=0}^{N-1} x(n) F_p \tag{2}$$

where u is the fractional-time domain samples and X_p is the p-th order FrFT of $x(n)$. FrFT can be interpreted as a rotation by an anticlockwise angle α in the time–frequency plane. The relationship between the rotation angle α and the transformation order p is $\alpha = \pi/2 * p$.

The FrGT of $x(n)$ can be expressed as (Akan and Önen, 2007)

$$GT = \sum_{t_0=1}^{N} x(n+n_0) g^*(n_0) F_p. \tag{3}$$

It is proved that the TFDs of Gabor transform satisfy the magnitude-wise shift invariance in time–frequency plane if the window function is Hermite–Gaussian function (Durak and Arikan, 2003). When we ignore the phase changes of transformational process, FrGT with clear physics meaning can be written as follows (Chen et al., 2013):

$$|GT| = \left| \sum_{t_0=1}^{N} x(n+n_0) g_p^*(n_0) \exp(-i2\pi n_0 m) \right| \tag{4}$$

where $g(n)$ is the window function, $g_p(n)$ is the p-th FrFT of $g(n)$, discrete window function of the length N is

$$g_p(n) = \exp\left(-\pi B_{X_p} n^2 / T_{X_p}\right) \cdot F_p, 0 \le n \le N-1 \tag{5}$$

where B_{X_p} and T_{X_p} are the bandwidth and time width of X_p.

2.2. Searching for the optimal order with kurtosis maximization

Optimal order of FrGT will directly affect the results of TFDs; hence the method of searching the optimal order of FrGT is important. Generally speaking, the signal's FrGT has the same optimal order with signal's FrFT. Kurtosis maximization is usually used to detect signal or contrast cost function. Spectral kurtosis maximization is used to detect small target in fractional domain (Guan et al., 2012). Kurtosis maximization method is used to construct cost function according to the property of signals which has a super-Gaussian distribution (Du and Kopriva, 2008; Li and Adali, 2008). In this paper, kurtosis maximization is introduced into fractional time–frequency analysis methods. Seismic data can be generally considered as signals with a super-Gaussian distribution (Walden, 1985), and the kurtosis sign is positive for super-Gaussian distribution. FrFT is linear transform (Almeida, 1994); the kurtosis sign is invariant by any linear transforms (Mansour and Jutten, 1999). The optimal order of FrFT can be given by kurtosis maximization

$$K_p = \frac{E\left\{\left(\left|X_p(u)\right| - \left|\overline{X}_p(u)\right|\right)^4\right\}}{E^2\left\{\left(\left|X_p(u)\right| - \left|\overline{X}_p(u)\right|\right)^2\right\}} \tag{6}$$

where $X_p(u)$ is the p-th order FrFT of signal, and $|X_p(u)|$ is the absolute value of $X_p(u)$, the TFDs are taken the absolute value in this paper, for consistency, the absolute value of FrFT is used for kurtosis maximization in fractional domain. $\overline{X}_p(u)$ is the mean of $X_p(u)$, and E is the expected value of variable. The maximal kurtosis corresponding order is the optimal order of FrFT.

$$p_{opt} = \arg \max_{0 \le p < 4} \left\{K_p\right\} \tag{7}$$

By using the properties $F_{-p}(t,u) = F_p^*(t,u)$ (Candan et al., 2000), the period of transform order p is 4, $F_{-p} = F_{4-p}$, since the absolute value of magnitude is only discussed in this paper, we obtain

$$\left|X_{-p}(u)\right| = \left|X_{4-p}(u)\right| = \left|X_p^*(u)\right| = \left|X_p(u)\right|. \tag{8}$$

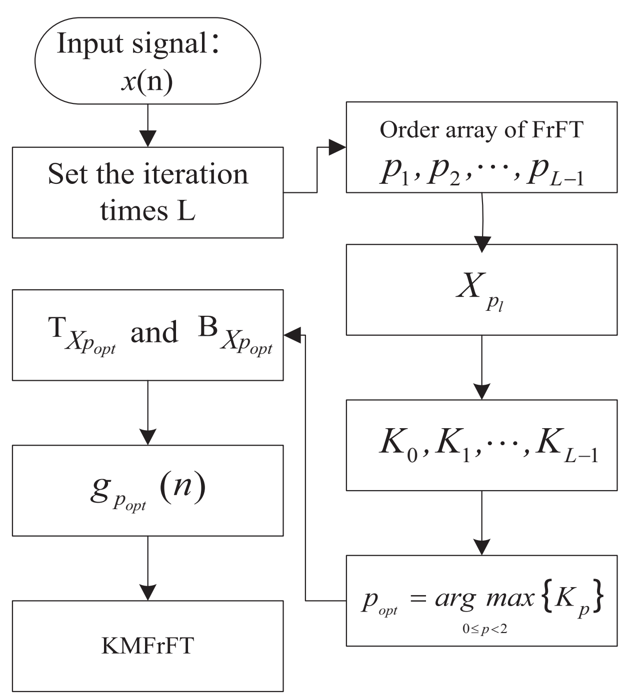

Fig. 1. Flowchart of KMFrGT.