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Comparison of two schemes for Laplace-domain 2D scalar wave equation

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ABSTRACT

Laplace-domain modeling plays an important role in Laplace-domain full waveform inversion. In order to provide efficient numerical schemes for Laplace-domain modeling, two 9-point schemes for Laplace-domain 2D scalar equation are compared in this paper. Compared to the finite-element 9-point scheme, the average-derivative optimal 9-point scheme reduces the number of grid points per pseudo-wavelength from 16 to 4 for equal directional sampling intervals. For unequal directional sampling intervals, the average-derivative optimal 9-point scheme reduces the number of grid points per pseudo-wavelength from 13 to 4. Numerical experiments demonstrate that the average-derivative optimal 9-point scheme is more accurate than the finite-element 9-point scheme for the same sampling intervals. By using smaller sampling intervals, the finite-element 9-point scheme can approach the accuracy of the average-derivative optimal 9-point scheme, but the corresponding computational cost and storage requirement are much higher.

1. Introduction

Using the zero frequency component of the damped wavefield, Laplace-domain full waveform inversion (FWI) can yield a smooth velocity model which can be used as a starting model for subsequent frequency-domain full waveform inversion (Shin and Cha, 2008). Because of its less sensitivity to the lack of low-frequency component, Laplace-domain FWI has been successfully applied to real data (Ha et al., 2012; Shin et al., 2010). Forward modeling in Laplace-domain is an essential part of Laplace-domain FWI. Therefore, it is important to make comparisons between different Laplace-domain schemes and provide efficient schemes for Laplace-domain FWI.

Laplace-domain schemes can be directly obtained from frequency-domain schemes. Frequency-domain schemes for 2D scalar wave equation include the classical 5-point scheme (Pratt and Worthington, 1990), the optimal 9-point scheme for equal directional sampling intervals (Jo et al., 1996; Operto et al., 2007), the average-derivative optimal scheme (Chen, 2012), and the directional-derivative method (Chen, 2013). However, the dispersion analysis of Laplace-domain schemes is different from that of frequency-domain schemes. Shin et al. (2002) developed a method to perform Laplace-domain numerical dispersion analysis by expressing Laplace-domain dispersion relation as the square root of the ratio of numerical eigenvalue to analytical eigenvalue. However, this dispersion relation depends on damping constant, velocity, and sampling interval as well as propagation angle. Therefore, it is difficult to draw a general conclusion and to optimize the scheme. Based on the skin depth in Laplace-domain acoustic wave equation (Um et al., 2012), Chen (2014) developed a Laplace-domain method of numerical dispersion analysis by defining a pseudo-wavelength as 2π times the skin depth. The dispersion relation can be expressed as a normalized attenuation propagation velocity which depends on the number of grid points per pseudo-wavelength as well as propagation angle.

In this paper, we use the method in Chen (2014) to make comparisons between two 9-point schemes for 2D Laplace-domain scalar wave equation. In the next section, we will present the Laplace-domain average-derivative optimal 9-point scheme and the finite-element 9-point scheme. This is followed by comparisons between the two schemes in terms of numerical dispersion analysis. Numerical examples are then presented to demonstrate the theoretical analysis.

2. Two Laplace-domain 9-point schemes

Consider the two-dimensional (2D) scalar wave equation in Laplace domain (Shin et al., 2002):

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial z^2}-\frac{s^2}{v^2}P=0, \qquad (1)$$

where P is the pressure wavefield, the real number s is the Laplace damping constant, and $v(x, z)$ is the velocity.

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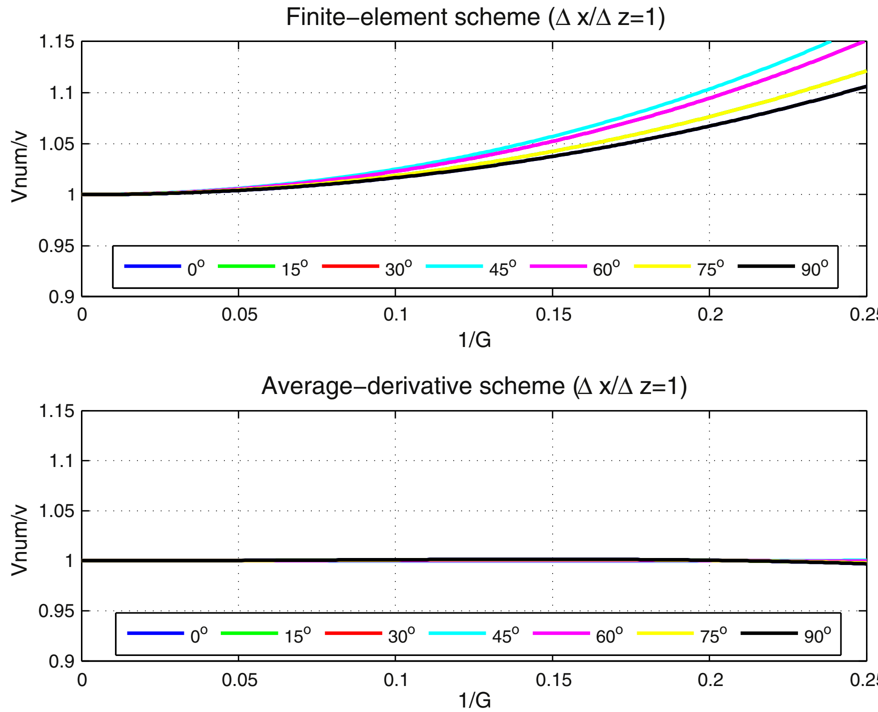

Fig. 1. Normalized numerical attenuation propagation velocity of the finite-element 9-point scheme (5) and the average-derivative optimal 9-point scheme (2) for different propagation angles for the case when $\frac{\Delta x}{\Delta z} = 1$.

Based on the frequency-domain scheme developed in Chen (2012), one can obtain an average-derivative 9-point scheme for Eq. (1):

$$\begin{aligned}&\frac{\overline{P}_{m+1,n}-2\overline{P}_{m,\,n}+\overline{P}_{m-1,\,n}}{\Delta x^2}+\frac{\widetilde{P}_{m,\,n+1}-2\widetilde{P}_{m,\,n}+\widetilde{P}_{m,\,n-1}}{\Delta z^2}\\&-\frac{s^2}{v_{m,\,n}^2}\Big[cP_{m,\,n}+d\big(P_{m+1,\,n}+P_{m-1,\,n}+P_{m,\,n+1}+P_{m,\,n-1}\big)\\&+b\big(P_{m+1,\,n+1}+P_{m-1,\,n+1}+P_{m+1,\,n-1}+P_{m-1,\,n-1}\big)\Big]=0,\end{aligned}\tag{2}$$

where

$$\overline{P}_{m+j,n}=\frac{1-\alpha}{2}P_{m+j,n+1}+\alpha P_{m+j,n}+\frac{1-\alpha}{2}P_{m+j,n-1},\quad j=1,0,-1,\tag{3}$$

$$\widetilde{P}_{m,n+j}=\frac{1-\beta}{2}P_{m+1,n+j}+\beta P_{m,n+j}+\frac{1-\beta}{2}P_{m-1,n+j},\quad j=1,0,-1.\tag{4}$$

Here $P_{m,n} \approx P(m\Delta x, n\Delta z)$, $v_{m,n} \approx v(m\Delta x, n\Delta z)$, Δx and Δz are sampling intervals in x- and z-directions, respectively, α, β, c and d are weighted coefficients which should be optimized, and $b = \frac{1-c-4d}{4}$ (Chen, 2014).

Based on finite-element formulation, Shin et al. (2002) derived the following Laplace-domain finite-element 9-point scheme:

$$\begin{aligned}&\frac{1}{6}\frac{1}{\Delta x^2}\left[P_{m+1,\,n-1}-2P_{m,\,n-1}+P_{m-1,\,n-1}\right]\\&+\frac{2}{3}\frac{1}{\Delta x^2}\left[P_{m+1,\,n}-2P_{m,\,n}+P_{m-1,\,n}\right]\\&+\frac{1}{6}\frac{1}{\Delta x^2}\left[P_{m+1,\,n+1}-2P_{m,\,n+1}+P_{m-1,\,n+1}\right]\\&+\frac{1}{6}\frac{1}{\Delta z^2}\left[P_{m-1,\,n+1}-2P_{m-1,\,n}+P_{m-1,\,n-1}\right]\\&+\frac{2}{3}\frac{1}{\Delta z^2}\left[P_{m,\,n+1}-2P_{m,\,n}+P_{m,\,n-1}\right]\\&+\frac{1}{6}\frac{1}{\Delta z^2}\left[P_{m+1,\,n+1}-2P_{m+1,\,n}+P_{m+1,\,n-1}\right]-\frac{s^2}{v_{m,\,n}^2}P_{m,\,n}=0.\end{aligned}\tag{5}$$

Note that the scheme (5) is a special case of the scheme (2). If one takes $\alpha = \beta = \frac{2}{3}$, $c = 1$, and $d = 0$, then the scheme (2) becomes the scheme (5).

3. Comparison between two 9-point schemes

Consider an attenuating function in the following form

$$P(k,r)=P_0e^{-kr},\tag{6}$$

where $r = \sin(\theta)x + \cos(\theta)z$, P_0 is the amplitude at $r = 0$, θ is the propagation angle, and k is the pseudo-wavenumber.

Substituting Eq. (6) into Eq. (2) and assuming a constant v, one obtains the discrete dispersion relation

$$\frac{V_{num}}{v}=\frac{G}{2\pi}\sqrt{\frac{A}{B}},\tag{7}$$

where

$$\begin{aligned}A=&\left[(1-\alpha)\cosh\left(\frac{2\pi\cos(\theta)}{RG}\right)+\alpha\right]\left[2\cosh\left(\frac{2\pi\sin(\theta)}{G}\right)-2\right]\\&+R^2\left[(1-\beta)\cosh\left(\frac{2\pi\sin(\theta)}{G}\right)+\beta\right]\left[2\cosh\left(\frac{2\pi\cos(\theta)}{RG}\right)-2\right],\\B=&\,c+2d\left[\cosh\left(\frac{2\pi\cos(\theta)}{RG}\right)+\cosh\left(\frac{2\pi\sin(\theta)}{G}\right)\right]\\&+4b\cosh\left(\frac{2\pi\cos(\theta)}{RG}\right)\cosh\left(\frac{2\pi\sin(\theta)}{G}\right),\end{aligned}$$

where $V_{num} = \frac{s}{k}$ is the numerical propagation velocity of attenuation, $G = \frac{2\pi}{k\Delta x}$ is the number of grid point per pseudo-wavelength, and $R = \frac{\Delta x}{\Delta z}$. Here, we only consider the case when $\Delta x \geq \Delta z$. The case when $\Delta x < \Delta z$ can be discussed similarly (Chen, 2014).

For the scheme (5), its discrete dispersion relation is a special case of Eq. (7) where $\alpha = \beta = \frac{2}{3}$, $c = 1$, and $d = 0$.