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# 3D stochastic inversion of potential field data using structural geologic constraints

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ABSTRACT

We introduce a new method to include structural orientation constraints into potential field inversion using a stochastic framework. The method considers known geological interfaces and planar orientation data such as stratification estimated from seismic surveys or drill hole information.

Integrating prior geological information into inversion methods can effectively reduce ambiguity and improve inversion results.

The presented approach uses cokriging prediction with derivatives. The method is applied to two synthetic models to demonstrate its suitability for 3D inversion of potential field data. The method is also applied to the inversion of gravity data collected over the Lalor volcanogenic massive sulfide deposit at Snow Lake, Central Manitoba, Canada. The results show that using a structurally-constrained inversion leads to a better-resolved solution.


## 1. Introduction

A variety of techniques have been utilized to invert potential field data (Oldenburg and Pratt, 2007). In most inversion applications, the problems are underdetermined and have non-unique solutions, i.e., there are an infinite number of models that can reproduce the geophysical observations. The purpose of any well-founded inversion method is to provide a model that is consistent with all the available geophysical, petrophysical and geological information. Many strategies can be used to deal with the non-uniqueness problem in gravity and magnetic inversion. They all involve constraints or regularization in order to limit the resulting solution space.

In a deterministic framework, Li and Oldenburg (2000) and Lelièvre and Oldenburg (2009) investigated options for incorporating structural orientation data into underdetermined inversions by minimization of an objective function. Barbosa and Silva (2006) introduced a technique in a user-friendly environment for helping forward modeling and testing geological hypotheses. Fullagar et al. (2008) applied an adaptive mesh for inversion of potential field anomalies. The method has the benefit of both reducing the size of the inversion problem and also incorporating geologic boundary information into the inversion.

Stochastic inversion methods (e.g., Eidsvik et al., 2004; Franklin, 1970; Mosegaard and Tarantola, 1995; Tarantola, 2005) have proven useful for many inversion problems. Bosch and McGaughey (2001), Calcagno et al. (2008) and Guillen et al. (2008) introduce a stochastic inversion framework that attempts to directly recover rock types. For these inversion methods known as lithological inversions, the model space is explored through a random walk process (Mosegaard and Tarantola, 1995). The advantage of these methods is that they allow us to assess model uncertainty but at a heavy computational cost.

Herein, we propose a stochastic inversion method that integrates structural orientation data (i.e. strike, dip) into the inversion process. One of the stochastic methods that has been applied in the inversion of geophysical data is the geostatistical approach. The geostatistical approach has been successfully applied by Dong (1990), Haas and Dubrule (1994), Torres-Verdin et al. (1999), Chasseriau and Chouteau (2003), Gloaguen et al. (2005), Hansen et al. (2006), and Giroux et al. (2007). Shamsipour et al. (2010, 2011b, 2012) presented a geostatistical framework to use cokriging and conditional simulation for gravity, magnetic and joint 3D inversion of potential field data.

Most of the geostatistical methods are established on the stationary assumption. However, Shamsipour et al. (2013) introduced a method of inversion based on a geostatistical approach using non-stationary covariances. They succeeded in adding structural orientations by adjusting different variograms for segments with different structural orientations. To include a variable structure we propose to augment the inversion using cokriging with gradient information derived from ancillary geological or geophysical data, such as 3D seismic surveys. The approach is inspired by the work of Lajaunie et al. (1997) and Chilès and Delfiner (2012) who in the implicit modeling of geological surfaces by interpolation, added gradient data to the co-kriging system, which they named the potential-field interpolation method. The paper is organized as follows: first, we show how gradient constraints can be

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incorporated in the geostatistical prediction. Then we review the formulation of the inverse problem as a linear cokriging system and expand this formulation to include gradient constraints from structural orientation data. Then, applications of the inversion algorithm on synthetic examples are presented to demonstrate the effectiveness of including gradient constraints in the inversion. Thereafter we present and discuss results from the application of the constrained inversion method on a multi-parameter geophysical survey (i.e. gravity and 3D seismic data) acquired over the Lalor massive sulfide deposit in Central Manitoba, Canada. Then the applicability of our method is discussed before we finish with conclusions.

## 2. Theory

In this section, we first explain cokriging prediction with derivatives and then in the next section, we extend this approach to linear stochastic inversion. Cokriging (Chilès and Delfiner, 2012) is a stochastic interpolation and extrapolation tool to improve the prediction of a primary variable at unsampled locations using the spatial correlation between secondary variables and the primary variable. Suppose that we want to predict the primary variable $Z_1(x_0)$ from realizations of secondary random functions $Z_i(x_\alpha)$ known over sample sets $S_i = \{x_\alpha : Z_i(x_\alpha)$ known$\}$. We can write the cokriging prediction (Chilès and Delfiner, 2012) as

$$\hat{Z} = \sum_i \sum_{\alpha \in S_i} \lambda_{i\alpha} Z_i(x_\alpha) \tag{1}$$

where $x_\alpha$ is used for generic data points, but the sample sets $S_i$ are in general different for different indices representing primary and secondary variables.

Now suppose that we consider prediction of the primary variable using its derivatives. In this case the cokriging formula can be written as

$$\hat{Z} = \sum_{x_\alpha \in S_1} \lambda_{1\alpha} Z(x_\alpha) + \sum_{x_\beta \in S_2} \lambda_{2\beta} \frac{\partial Z}{\partial u_\beta}\left(x_\beta\right) + \sum_{x_\beta \in S_3} \lambda_{3\beta} \frac{\partial Z}{\partial v_\beta}\left(x_\beta\right) + \sum_{x_\beta \in S_4} \lambda_{4\beta} \frac{\partial Z}{\partial w_\beta}\left(x_\beta\right) \tag{2}$$

where $u$, $v$ and $w$ are the directional derivatives and $x_\beta$ is the secondary variable. Normally, $u$, $v$ and $w$ are the directions of the coordinate axes. For simplicity, we replace

$$\frac{\partial Z}{\partial x}(x_i) = G_i^x,$$

$$\frac{\partial Z}{\partial y}(x_i) = G_i^y,$$

$$\frac{\partial Z}{\partial z}(x_i) = G_i^z.$$

In order to solve the cokriging system, we need all the covariances and cross-covariances involved in the above system. They can all be expressed based on $C_{ZZ}$, the auto-covariance of Z. Assume there are two locations $x_a$ and $x_b$ and define $h = (h_x, h_y) = x_a - x_b$ and $r = |h|$, then the required covariances can be written as (Blanc-Lapierre and Fortet, 1953; Chilès and Delfiner, 2012)

$$C_{ZG^x}(x_a, x_b) = \mathrm{Cov}\left(Z(x_a), Z'_x(x_b)\right) = \frac{h_x}{r} C'_{ZZ} \tag{3}$$

$$C_{G^xG^y}(x_a, x_b) = \mathrm{Cov}\left(Z'_x(x_a), Z'_y(x_b)\right) = \frac{h_x h_y}{r^2}\left(\frac{1}{r} C'_{ZZ} - C''_{ZZ}\right) \tag{4}$$

$$C_{G^xG^x}(x_a, x_b) = \mathrm{Cov}\left(Z'_x(x_a), Z'_x(x_b)\right) = \left(\frac{h_x^2}{r^3} - \frac{1}{r}\right) C'_{ZZ} - \left(\frac{h_x}{r}\right)^2 C''_{ZZ} \tag{5}$$

where $Z'$ and $C'$ are the first derivative of the random function and the auto-covariance respectively. The second derivative of the auto-covariance is shown by $C''$. We have to mention here that common variogram functions such as the spherical, the exponential and the linear models are not differentiable. This is because of their singularity at the origin. In such cases, a differentiable variogram such as a Cauchy gravimetric and cubic is preferred.

From a mathematical point of view a derivative is point support information, however in reality the information used for geological imaging is not a point derivative but rather a gradient defined over a finite support. Renard and Ruffo (1993) modeled the derivative information using a Gaussian weighting function with main axis lengths, which these axes can be set random with reasonable values. In the next section we will explain another method to model derivative information by finite increments. In this method the derivative information is not a point derivative but rather an average over some support.

### 2.1. Point pairs method

The method of point pairs represents the gradient of a random function by a pair of points (Delhomme, 1979). We use point pairs $x_\beta$ (defining zero gradients) as a secondary variable in the prediction to incorporate structural geologic constraints in the cokriging system. Note that these point pairs define gradients but not the values themselves. The cokriging system can now be written as (Chilès and Delfiner, 2012)

$$\hat{Z} = \sum_\alpha \lambda_{1\alpha} Z(x_\alpha) + \sum_\beta \lambda_{2\beta}\left[Z\left(x_\beta + hu_\beta\right) - Z\left(x_\beta - hu_\beta\right)\right] \tag{6}$$

where $u$ is the unit vector, $u \in R^n$, and $h$ is the distance. It should be noted, in cases where the difference in values over point pairs is zero, the contribution of the second term in Eq. (6) is not zero (Brochu and Marcotte, 2003; Chilès and Delfiner, 2012). Hence, the weights ($\lambda_{1\alpha}$) are different when we use kriging based on the $Z(x_\alpha)$ (physical properties) alone, even though the contribution of the increment data vanishes in the kriging result, their information contributes to the final estimate because they are taken into account by the kriging equations.

Chilès and Delfiner (2012) reported that the pair point distances should be equal to the grid cell width to obtain adequate and robust results. The selected pair points must not be located too close to each other or adjacent to existing observations to prevent numerical instability in the cokriging system.

Lajaunie et al. (1997) applied the same method to interpolate geological interfaces in the presence of orientation data. The method assumes that geological interfaces are iso-surfaces of a scalar 3D potential field. In their study they used three different types of data: gradient data, data on tangents and increments. Their method has no direct measurement of the random function and the major drawback is to predict the covariance model.

## 3. Inversion

Shamsipour et al. (2010) presented a stochastic framework for the inversion of potential field data using cokriging. This method can be applied to any linear geophysical case. Consider that there are $n$ data observations, $d$, and m parameters (physical properties of rectangular prisms) $p$, their relationship can be written in a matrix form:

$$d_{n\times 1} = A_{n\times m} p_{m\times 1} \tag{7}$$