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The mutual inductance calculation between circular and quadrilateral coils at arbitrary attitudes using a rotation matrix for airborne transient electromagnetic systems

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ABSTRACT

Performance testing and calibration of airborne transient electromagnetic (ATEM) systems are conducted to obtain the electromagnetic response of ground loops. It is necessary to accurately calculate the mutual inductance between transmitting coils, receiving coils and ground loops to compute the electromagnetic responses. Therefore, based on Neumann's formula and the measured attitudes of the coils, this study deduces the formula for the mutual inductance calculation between circular and quadrilateral coils, circular and circular coils, and quadrilateral and quadrilateral coils using a rotation matrix, and then proposes a method to calculate the mutual inductance between two coils at arbitrary attitudes (roll, pitch, and yaw). Using coil attitude simulated data of an ATEM system, we calculate the mutual inductance of transmitting coils and ground loops at different attitudes, analyze the impact of coil attitudes on mutual inductance, and compare the computational accuracy and speed of the proposed method with those of other methods using the same data. The results show that the relative error of the calculation is smaller and that the speed-up is significant compared to other methods. Moreover, the proposed method is also applicable to the mutual inductance calculation of polygonal and circular coils at arbitrary attitudes and is highly expandable.

1. Introduction

The testing and the calibration of the performance of airborne transient electromagnetic (ATEM) systems involve paving ground loops to simulate finite conductors. Fig. 1 shows the exploration schematic diagram using a fixed-wing ATEM system and a helicopter ATEM system. During the tests, the ground loops are fixed to their positions, and airplanes carrying the ATEM systems fly over the ground loops for the measurement. Fitterman (1998) adopted a wire loop to calibrate the geometric errors of airplanes during flight. Davis and Macnae (2008) used multi-turn large loops to calibrate the geometric errors of five ATEM systems, including VTEM, HoistEM, AeroTEM, TEMPEST and SkyTEM. In addition, they performed quantitative tests of transmitting current waveforms. Yin and Hodges (2009) tested the performance of the HeliGEOTEM electromagnetic system of Fugro. Ji et al. (2010, 2011a,b) tested the helicopter ATEM system developed by Jilin University using ground loops. The testing and the calibration of electromagnetic systems require the calculation of induced current responses in the ground loops. The key to calculate this current is to accurately determine the mutual inductance among the transmitting coils, receiving coils and ground loops. In actual flight tests, the attitudes of the transmitting coils and receiving coils change constantly due to variations in the velocity, air flow, wind direction and other factors. Therefore, it is necessary to calculate the mutual inductance of two coils at arbitrary attitudes for the sake of accurately calculating the induced currents in the ground loops.

Currently, the mutual inductance of two coils is commonly calculated by Neumann, Maxwell and Grover formulas (Bueno and Assis, 1997). The calculation methods for the mutual inductance between two parallel coaxial and non-coaxial circular coils, hexagonal coils as well as circular and quadrilateral coils are relatively mature (Akyel et al., 2009; Conway, 2007, 2010; Pankrac, 2011; Sonntag et al., 2008; Xiang, 2000). To calculate the mutual inductance between two non-parallel coils, Hannakam and Nolle (1980) used the relationship between the position vectors of two circular loops and Neumann inductance formula to calculate the inductance of circular coils at arbitrary positions. Hannakam and Tepe (1981) calculated the mutual inductance of two segments based on known coordinates of the vertices of polygonal loops and then performed an algebraic summation to calculate the mutual inductance between two arbitrary polygon loops. As for the numerical calculation of the mutual inductance of two non-parallel coils, Davis (2007) and Dumitru (2002) calculated the mutual inductance of two inclined segments by applying the vector dot product and cross product calculation. This polygonal mutual inductance algorithm is not applicable to the mutual inductance calculation of two coils in parallel or vertical attitudes. Babic and Akyel (2008) calculated the mutual inductance of unidirectional inclined circular coils based on Neumann's formula. Babic et al. (2009) and Babic et al. (2010) determined all

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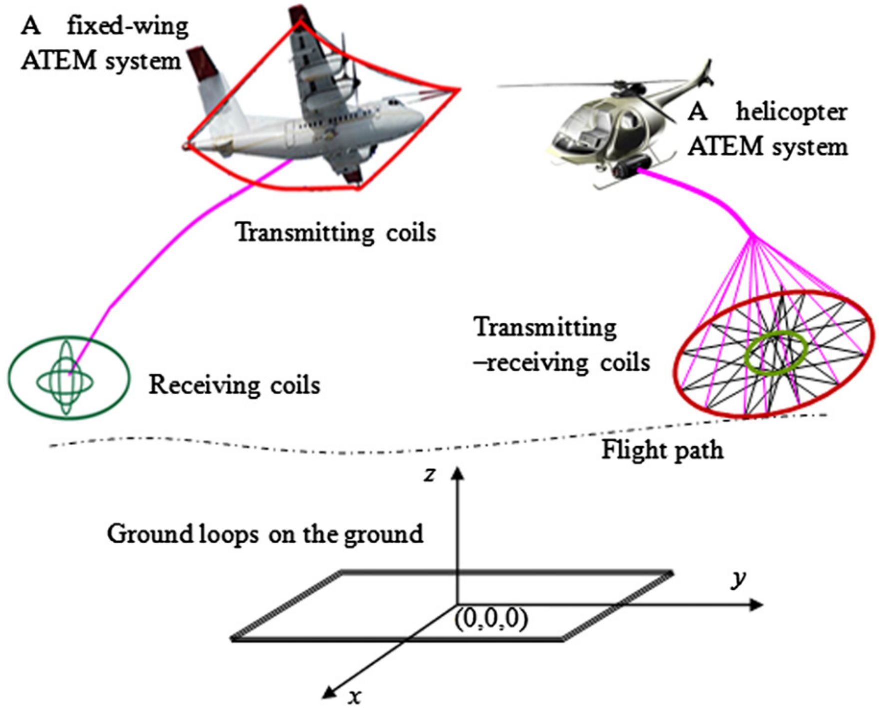

Fig. 1. Schematic diagram of tests for ATEM systems of a helicopter and of a fixed-wing airplane over ground loops.

parameters of the plane equation by establishing the plane equation of circular coils after rotation and calculated the mutual inductance of two coils in arbitrary attitudes and positions. This method is applicable to circular coils and coils of simple shapes; however, it is rather complex and difficult to establish a plane equation and to determine the plane parameters for arbitrary shapes of polygonal loops. Ansoft, COMSOL Multiphysics and other commercial software can also be adopted to calculate the mutual inductance of polygonal loops under arbitrary attitudes (Engel and Rohe, 2006; Ghali and Rahman, 2009). Most of these systems are based on finite elements and boundary elements, so, during the calculation, the number of dissection networks is increased to improve the computational accuracy, leading to a low computational speed and a long time, especially when calculating the mutual inductance of any two large coils (with edge length of several meters) at arbitrary attitudes. If a small workstation is employed, it takes more than 24 h to calculate the mutual inductance of the parallel and coaxial coils, letting alone calculating that of arbitrary attitudes. The result is inefficient and unable to meet the needs of multipoint mutual inductance calculation under arbitrary attitudes.

During flight tests of ATEM systems, the attitudes of the transmitting and receiving coils change constantly, making it difficult to obtain the shortest distance between the coils or the point coordinates of the coils, so, it is impossible to use the Hannakam's and Tepe's polygonal mutual inductance algorithm directly. However, it is common to measure the attitude variations of ATEM systems directly or indirectly by installing an inertial navigation system or two laser altimeters, differential Global Position System (GPS) devices and inclinometers (Gunnink et al., 2012; Hefford et al., 2006; Smith, 2001), and then we can use the Neumann mutual inductance formula and the measured attitude variations to transform the coordinates of circular coils or polygonal coils at arbitrary attitudes with a rotation matrix. Thus the singular values of two vertical coils can be calculated as well as the mutual inductance of polygonal and circular coils at arbitrary attitudes and arbitrary positions.

Based on Neumann's formula, this study transforms the coordinates of circular coils or quadrilateral coils under arbitrary attitudes by using rotation matrix, and calculates the mutual inductance of quadrilateral and circular coils by combining numerical integration of loop. As for the mutual inductance calculation of circular and circular coils at arbitrary attitudes, we estimate the computational accuracy of the proposed method by comparing its results with those ones obtained with the algorithm from Babic et al. (2010). As for the mutual inductance calculation of quadrilateral coils at arbitrary attitudes, we verify the computational speed of the proposed method by comparing its results with those ones obtained with the Ansoft.

This paper is organized as follows. Section 2 details the mutual inductance calculation of circular and quadrilateral coils at arbitrary attitudes. Section 3 details that of quadrilateral coils. In Section 4, some typical numerical examples are given to verify the proposed method. Finally, Section 5 summarizes the paper.

2. Mutual inductance calculation of circular and quadrilateral coils at arbitrary attitudes

Suppose that coil I is quadrilateral and coil II refers to a circular coil, as shown in Fig. 2. Then, we establish the xyz Cartesian coordinate system as shown for coil I. $O(0,0,0)$ is the origin of plane xOy. When coil II is parallel to coil I, establish the $x''y''z''$ rectangular coordinate system for coil II and when the attitude of coil II has changed, establish the $x'y'z'$ Cartesian coordinate system. Fig. 2 shows the attitude of coil II rotation around the x'', y'', and z'', respectively. The coordinates of the vertices of coil I are $P_{11}(x_{11}, y_{11}, z_{11})$, $P_{12}(x_{12}, y_{12}, z_{12})$, $P_{13}(x_{13}, y_{13}, z_{13})$ and $P_{14}(x_{14}, y_{14}, z_{14})$. The radius of coil II is r_2 and the center coordinate is, O_c (x_c, y_c, z_c) with the center coordinate changing dynamically during flight.