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Volume Continuation of potential fields from the minimum-length solution: An optimal tool for continuation through general surfaces

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ABSTRACT

Many methods have been used to upward continue potential field data. Most techniques employ the Fast Fourier transform, which is an accurate, quick way to compute level-to-level upward continuation or spatially varying scale filters for level-to-draped surfaces. We here propose a new continuation approach based on the minimum-length solution of the inverse potential field problem, which we call *Volume Continuation* (VOCO). For real data the VOCO is obtained as the regularized solution to the Tikhonov problem. We tested our method on several synthetic examples involving all types of upward continuation and downward continuation (level-to-level, level-to-draped, draped-to-level, draped-to-draped). We also employed the technique to upward continue to a constant height (2500 m a.s.l.), the high-resolution draped aeromagnetic data of the Ischia Island in Southern Italy. We found that, on the average, they are consistent with the aeromagnetic regional data measured at the same altitude. The main feature of our method is that it does not only provide continued data over a specified surface, but it yields a volume of upward continuation. For example, the continued data refers to a volume and thus, any surface may be easily picked up within the volume to get upward continuation to different surfaces. This approach, based on inversion of the measured data, tends to be especially advantageous over the classical techniques when dealing with draped-to-level upward continuation. It is also useful to obtain a more stable downward continuation and to continue noisy data. The inversion procedure involved in the method implies moderate computational costs, which are well compensated by getting a 3D set of upward continued data to achieve high quality results.

1. Introduction

Upward continuation is used to transform anomalies measured at a surface into anomalies that would be measured at some higher altitude surface. It can also be used to merge data at different altitudes, such as, e.g., those measured on irregular surfaces (i.e. in case of draped airborne surveys) and to continue the whole dataset to a given surface (e.g., Paoletti et al., 2004, 2005; Paterson et al., 1990; Pilkington and Roest, 1992; Ridsdill-Smith, 2000). This transformation is also helpful to enhance the effects of deep sources, as it attenuates the highest frequency content of the signal, which is usually associated to shallow sources. Finally, multiscale methods such as the continuous wavelet transform (Fedi and Cascone, 2011; Fedi et al., 2010; Mauri et al., 2010; Sailhac et al., 2009), the DEXP transformation (Fedi, 2007) and the multiridge analysis (Cella et al., 2009) involve potential field data available on a 3D volume, which in turn is generated by upward continuation of data measured at a single – flat or draped – surface.

Upward continuation originates from Green's third identity (Blakely, 1996), which states that if U is a harmonic continuous function, with continuous derivatives through a regular region R, then its value at any point P within the harmonic region R, can be evaluated from its behaviour on the boundary S:

$$U(P) = \frac{1}{4\pi}\int_S \left(\frac{1}{r}\frac{\partial U}{\partial n} - U\frac{\partial}{\partial n}\frac{1}{r}\right) dS \tag{1}$$

where n is the outward normal direction, r the distance from P to the point of integration on S. No information about the source is needed except that it is not located within the region R.

In the following, we will describe the four types of performing upward continuation: level-to-level, level-to-draped, draped-to-level and draped-to-draped. The simplest case is the level-to-level upward continuation, when the potential field data is measured on a constant altitude surface z_0 and continued to some higher altitude plane. The process is defined by an integral formulation:

$$U(x,y,z_0-\Delta z) = \frac{\Delta z}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{U(x',y',z_0)}{\left[(x-x')^2+(y-y')^2+\Delta z\right]^{3/2}} dx'dy' \tag{2}$$

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where $\Delta z > 0$, and z is assumed negative outward. Eq. (2) is a convolution integral and can be solved via the Fourier transform and the convolution theorem. The numerical implementation of this formula obviously considers a finite-extent dataset and equally spaced data, which leads to known types of errors for the continued data (Fedi et al., 2012). Upward continued data can be calculated by convolution in either the space domain or the Fourier domain. In this last domain, the Fourier transform of the data is simply multiplied by the frequency operator:

$$e^{-|\mathbf{k}|\Delta z} \quad \Delta z>0 \tag{3}$$

where $\mathbf{k}$ is the wavenumber vector. As said before, real data is discrete and refers to a finite survey area. Thus, when using circular convolution to calculate upward continuation in the frequency domain, aliasing errors can affect the low-frequency content of upward continued data. These errors can be reduced by performing the Fourier transform on a larger dataset, which extends outside the survey area (Fedi et al., 2012; Oppenheim and Schafer, 1975). The enlarged dataset may be built utilizing data from other surveys or through extrapolation algorithms: zero-padding and symmetric extension (Fedi et al., 2012), maximum entropy extension (Gibert and Galdéano, 1985) and others.

Eqs. (2) and (3) are strictly valid for level-to-level continuations; however potential field literature is rich in algorithms performing upward continuation between uneven surfaces. Among them, Cordell (1985), Pilkington and Roest (1992) and Ridsdill-Smith (2000) developed algorithms for level-to-draped upward continuation. Whereas, Xia et al. (1993), Meurers and Pail (1998), Fedi et al. (1999), Ridsdill-Smith (2000) and Wang (2006) formulated upward continuation algorithms for draped-to-level surfaces.

In this paper we define a new approach that performs upward continuation using the relation established by Cribb (1976) between the minimum-length solution of the inverse potential field problem and upward continuation. We call this method Volume Upward Continuation (VOCO) as this approach has the advantage of generating the field as a unique solution in a 3D volume, in which all four types of continuations (level-to-level, level-to-draped, draped-to-level and draped-to-draped) are naturally defined.

As known, inversion implies higher computational costs than standard continuation algorithms. They are however, well compensated by the versatility of the method in providing from a single inversion, the continuation along any desired surface inside the source volume and by the high-quality of the continuation.

2. Volume Upward Continuation of potential fields

2.1. Theoretical background

As described in the following, the Volume Upward Continuation of a magnetic field is computed by: i) inverting the magnetic anomaly to obtain the minimum-length solution; and ii) converting the magnetization volume into an upward continued field volume.

The inverse problem for potential fields (e.g., Fedi et al., 2005) can be formulated by the following integral:

$$f(\mathbf{r}) = \int_V K(\mathbf{r},\mathbf{r}_0)M(\mathbf{r}_0)d\mathbf{r}_0^3 \tag{4}$$

where $f(\mathbf{r})$ is the potential field measured at the point $\mathbf{r}$, $M(\mathbf{r}_0)$ is the unknown source distribution in the volume Ω and $K(\mathbf{r}, \mathbf{r}_0)$ is the kernel. In the case of the magnetic field we will denote $f = \Delta T$ and consider K as the field measured at a distance $\mathbf{r}$ given by a magnetic dipole with unit-magnetization at the point $\mathbf{r}_0$.

The kernel K can be expressed as:

$$K(\mathbf{r},\mathbf{r}_0) = \frac{\mu_0}{4\pi}\frac{\partial^2}{\partial n \partial t}\frac{1}{\|\mathbf{r}-\mathbf{r}_0\|_2} \tag{5}$$

where μ_0 is the magnetic permeability of free space, $\mathbf{t}$ and $\mathbf{n}$ are unit-vectors along the inducing field and magnetization vectors, and $||\mathbf{r} - \mathbf{r}_0||_2$ denotes the Euclidean norm of $\mathbf{r} - \mathbf{r}_0$.

Discretizing the volume Ω in an $N_x \times N_y \times N_z$ grid of blocks, assuming that the magnetization is a piecewise function constant in each cell Ω_j, we can write a linear system of equations, calculating expression (5) in each cell. In this way we will get $P = N_x \times N_y$ equations (one for each measurement point). Every equation will then have $N = N_x \times N_y \times N_z$ unknowns leading to the linear system:

$$\mathbf{Km} = \mathbf{d} \tag{6}$$

where $\mathbf{K}$ is the kernel coefficients matrix and $\mathbf{d}$ is the measured data vector. The coefficient matrix $\mathbf{K}$ has elements given by the expression:

$$K_{ij} = \int_{\Omega_j} K(\mathbf{r},\mathbf{r}_0)d\mathbf{r}_0^3 \qquad i = 1,\ldots, P, \qquad j = 1,\ldots,N. \tag{7}$$

The simplest solution of such an under-determined inverse problem is the one minimizing the Euclidean norm of the solution $\mathbf{m}$: $\left(\sum_{i=1}^{N}|m_i|^2\right)^{1/2}$. This is called minimum-length solution (Menke, 1989):

$$\mathbf{m} = \mathbf{K}^T\left(\mathbf{KK}^T\right)^{-1}\mathbf{d}. \tag{8}$$

In order to obtain an upward continued field volume, we can use the relation shown by Cribb (1976). For a vertical direction of both the inducing field and magnetization vectors, Cribb (1976) showed that the Fourier transform of $\mathbf{m}$ could be simply expressed as:

$$F[\mathbf{m}_i] = 4e^{-|\mathbf{k}|h_i}F[\mathbf{d}] \qquad i = 1,\ldots,L \tag{9}$$

where F denotes the Fourier transformation, L is the number of layers, $\mathbf{m}_i$ is the magnetization intensity vector of the ith layer, $\mathbf{h}_i$ is its depth and $\mathbf{k}$ is the wavevector with components k_x, k_y and $(k_x^2 + k_y^2)^{1/2}$. The important information contained in Eq. (9) is that the Fourier Transform of the ith layer of the magnetization distribution $\mathbf{m}_i$ is directly related to the upward continued field of $\mathbf{d}$, to a distance equal to the opposite of the layer depth: $z_i = -h_i$ (Fedi and Pilkington, 2012). Thus, we can obtain $\mathbf{d}$ by anti-transforming the second member of Eq. (9):

$$\mathbf{d}_{h_i} = \frac{\mathbf{m}_i}{4} \qquad i = 1,\ldots,L. \tag{10}$$

This shows that the field at the altitude $z_i = -h_i$ differs only for a numeric constant from the magnetization of the ith layer. We may therefore first compute the minimum-length solution (Eq. (8)), inserting $\mathbf{n} = \mathbf{t} = (0,0,1)$ in the kernel (Eq. (5)), then compute the Volume upward Continuation (VOCO) through Eq. (10). This procedure may therefore be viewed as an effective alternative to common techniques of upward continuation of the magnetic field.

Note that for real data, a regularized solution may be preferable in order to dampen the disturbing error-propagation of the data noise. This may be accomplished not by minimizing the model norm (as in the case of the solution in Eq. (8)), but using the Tikhonov regularization that takes the form:

$$\min_{\mathbf{m}}\left\{\|\mathbf{Km}-\mathbf{d}\|_2^2 + \lambda^2\|\mathbf{m}\|_2^2\right\} \tag{11}$$