Journal of Applied Geophysics 111 (2014) 356–363

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Seismic attenuation compensation by Bayesian inversion

Shoudong Wang [a,b,c], Huijuan Song [a,b,c,d,*], Dengfeng Yang [a,b,c,e]

[a] College of Geophysics and Information Engineering, China University of Petroleum, Beijing, China
[b] State Key Laboratory of Petroleum Resource and Prospecting, Beijing, China
[c] National Engineering Laboratory of Offshore Oil Exploration, Beijing, China
[d] Jianghan Branch of Sinopic Geophysical Corporation, Qianjiang, China
[e] The Research of China National Offshore Oil Corporation (China) Ltd. Shenzhen, Guangzhou, China

ARTICLE INFO

Article history:
Received 20 January 2014
Accepted 16 October 2014
Available online 25 October 2014

Keywords:
Attenuation compensation
Bayesian inversion
Q estimation
Prior information

ABSTRACT

As an effective method to improve seismic data resolution, attenuation compensation has been paid great attention. The popular method inverse Q-filter performs effectively in phase correction. But its energy compensation part is at an extraordinary discount because of its instability. By contrast, the compensation method based on inversion has a great advantage in algorithm stability. In this paper, the inversion process is combined with Bayesian principle so that the prior information we learned about the actual model can be used sufficiently. Here, we have an assumption that it is more reasonable to describe the reflectivities with sparse distribution. This information, in general, can be transferred to a sparse constraint of the object function. And compared with Tikhonov regularization method, it is proved to perform better in seismic resolution improvement. Meanwhile, it is insensitive to the error of Q value. Example of real data shows the validity of the method.

1. Introduction

Seismic wave experiences energy attenuation and velocity dispersion while propagating through the subsurface medium. These properties are proved to be two fundamental factors of seismic resolution reducing. To represent the absorption issue mathematically, Futterman (1962) derived a dispersion relationship by assuming that the Q model is frequency independent. This model obtained widespread applications in the researches of seismic compensation. And it is used as the attenuation model in this paper.

To improve the resolution of seismic data, attenuation compensation must be taken into consideration. Many experts engaged themselves in compensation problems. Hargreaves et al. (1991) propose an approach similar to Stolt migration by regarding the compensation process as a kind of inverse Q-filter. The revise for energy absorption and phase distortion can be performed with the theory of wave-field downward continuation. With this method, the phase distortion can be corrected efficiently, but amplitude compensation is ignored because of its instability. Wang (2002, 2003) points out that stability and efficiency are two general concerns of compensation problem. He introduces a stable factor into inverse Q-filter to control algorithm stability. And considering the computational efficiency, the compensation process is achieved in two steps. The wavefield of the surface record is first extrapolated to the top of the current layer. And then constant Q inverse filtering was applied in each layer. Based on this method, Zhang et al. (2007) extract the gain-constraint frequencies from the Gabor spectrum to deduce the gain-constraint amplitude. Yan and Liu (2009) implemented the inverse Q-filtering on pre-stack common shot PP- and PS-waves along the ray path. In these methods, however, the amplitude compensation is suppressed by the stable factor. Absolutely, the record of deep reflection cannot be compensated well. Zhang and Ulrych (2007) regard the deabsorption process as a time-variant deconvolution and performed it with least squares inversion which can compensate the seismic amplitude better. This method was implemented in time domain. Based on exploding reflector idea, Wang (2011) reduces the compensation problem to an inversion problem and achieves it by Tikhonov regularization. As the compensation process is performed in frequency domain, we can choose the frequency band that involved in the inversion. So this method is useful for the data with high frequency or monofrequent noises. Tests on this method show its stability in amplitude compensation. Based on this theory, we have further research and get some improvement on the method.

As inversion problems are always undetermined, we need a rule to help us to choose a proper answer from the large amount of solutions. Tikhonov regularization added a smoothness constraint to the objective function to improve the inversion stability. However, it will reduce the seismic resolution, especially for the data with high dominant frequency and low signal noise ratio (SNR). Here, we choose Bayesian method to solve this inversion problem. The Bayesian theory provides us a framework to combine the priori model information with the information contained in the data and build a more refined inversion function. To describe the sparseness of the reflectivities, we suggest that it follows Laplace distribution. Then with Bayesian inversion method, the seismic resolution can be improved sufficiently, and this method is insensitive to the error of the Q value.

* Corresponding author.
E-mail address: songhj_wish@163.com (H. Song).

http://dx.doi.org/10.1016/j.jappgeo.2014.10.012
0926-9851/© 2014 Published by Elsevier B.V.

2. The theory of compensation based on inversion

Wang (2011) proposed a compensation method which attributes the deabsorption problem to an inversion issue. He states the thought and deductive procedure in detail. Now, we just have the key steps being discussed.

Suppose that the seismic wavelet is $w(t)$:

$$w(t) = \int_{-\infty}^{\infty} \hat{w}(\omega)e^{i\omega t}d\omega. \tag{1}$$

$\hat{w}(\omega)$ is the frequency spectrum. This equation means that the seismic wavelet can be regarded as the superposition of a series of simple harmonic waves $\hat{w}(\omega)e^{i\omega t}$, in which ω is the frequency of the harmonic wave.

With the theory of exploding reflector, the zero-offset synthetic record can be expressed as:

$$s(t) = \int d\omega \int r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'}dt', \tag{2}$$

in which $r(t')$ is the reflector coefficient of the model.

Then, using Futterman model, Wang (2011) gets the absorbed record:

$$s(t) = \int d\omega \int r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt'. \tag{3}$$

where $\gamma \approx \frac{1}{\pi Q}$. $Q(t')$ is the Q-factor of the medium, and ω_0 is the reference frequency which is always regarded as the dominant frequency.

However, the derivation by Wang (2011) doesn't take random noises into consideration. To close to the real data, we represent the data as:

$$s(t) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt' + n(t) \tag{4}$$

in which $n(t)$ is the environment noises.

By Fourier transformation, $n(t)$ can be rewritten as:

$$n(t) = \int_{-\infty}^{+\infty} n(\omega)e^{i\omega t}d\omega, \tag{5}$$

where $n(\omega)$ is the frequency spectrum of the environment noises.

Replace $n(t)$ in Eq. (4) with (5):

$$s(t) = \int_{-\infty}^{\infty} e^{i\omega t}\left(\int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt' + n(\omega)\right)d\omega \tag{6}$$

Then the spectrum of the attenuated data is like:

$$\hat{s}(\omega) = \hat{w}(\omega)\int_{-\infty}^{\infty} r(t')e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt' + n(\omega). \tag{7}$$

Suppose the data for compensation is deconvolved, the frequency spectrum can be reduced to:

$$\hat{s}(\omega) = \int_{-\infty}^{\infty} r(t')e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt' + \hat{n}(\omega) \tag{8}$$

where $\hat{n}(\omega) = \frac{n(\omega)}{\hat{w}(\omega)}$.

Based on Eq. (8), we define

$$g(\omega, t') = e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}. \tag{9}$$

Then Eq. (8) can be rewritten as:

$$\hat{s}(\omega) = \int g(\omega, t')r(t')dt' + \hat{n}(\omega) \tag{10}$$

For each separated frequency ω_i, we rewrite the equation in discrete form:

$$\hat{s}(\omega_i) = \sum_j g\left(\omega_i, t_j'\right)r\left(t_j'\right)\triangle t' + \hat{n}(\omega_i); \qquad i = 1, 2, 3 \cdots N \tag{11}$$

$t_j'(j = 1 : M)$ is the sampling time of the model and $\triangle t'$ indicates the time sampling interval.

The process of attenuation compensation is to obtain the reflector coefficient $r(t')$ from the frequency spectrum $\hat{s}(\omega)$. It is an inversion problem in some extent, and the kernel matrix is $\mathbf{G}_{ij} = g(\omega_i, t_j')\triangle t'$.

Then we can find something in common between the theory mentioned above and the deabsorption method proposed by Zhang and Ulrych (2007). The similarity is that both methods concludes the compensation issue to an inversion problem while they have a big difference. Zhang did the seismic compensation in time domain and its kernel matrix **G** is composed of time-variant wavelets with amplitude attenuation only. And the required dispersive phase correction is applied on the traces before amplitude compensation. In our method, the kernel matrix **G** takes both energy absorption and frequency dispersion into account. Phase correction and energy compensation are done in one step. In addition, it is implemented in frequency domain which is convenient for us to choose the frequency band with higher SNR.

3. The Bayesian inversion

In order to discuss the inversion problem, we represent the solution (the sparse reflectivities) with vector **m** and the observed data with **d**. The data **d** is not the right response of vector **m** because the seismic data is mixed with noises. Meanwhile, the observed data which is discrete can never describe the continuous model completely. All these factors lead to the fact that infinity models can be found to fit the data.

As the discrete frequency spectrum of absorbed data is like:

$$\hat{s}(\omega_i) = \sum_j g\left(\omega_i, t_j'\right)r\left(t_j'\right)\triangle t' + \hat{n}(\omega_i) \tag{12}$$

Let $g_{i,j} = g(\omega_i, t_j')\triangle t'$, Eq. (12) can be rewritten as:

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \vdots \\ \hat{s}_N \end{pmatrix} = \begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,M} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,M} \\ \vdots & \vdots & & \vdots \\ g_{N,1} & g_{N,2} & \cdots & g_{N,M} \end{pmatrix}\begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{pmatrix} + \begin{pmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \vdots \\ \hat{n}_N \end{pmatrix} \tag{13}$$

$\mathbf{d} = (\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_N)^{\mathrm{T}}$ is the observed data, $\mathbf{m} = (r_1, r_2, \cdots, r_M)^{\mathrm{T}}$ is the solution of the inverse problem, and $\hat{\mathbf{n}} = (\hat{n}_1, \hat{n}_2, \cdots, \hat{n}_N)^{\mathrm{T}}$ represents the recorded environment noises, in which $\bullet^{\mathrm{T}}$ indicates the transposition of vector $\bullet$. $\mathbf{G} = (g_{i,j})_{N \times M}$ is the kernel matrix.

Zhang (2009) points out the noises **n** is normally distributed as $N(\mu = 0, \sigma_n^2)$, where σ_n^2 is the variance and μ is the mean of the probability density function (PDF) respectively.

The likelihood of the data is given by:

$$p(\mathbf{d}|\mathbf{m}, \sigma_n) = \left(\frac{1}{2\pi\sigma_n^2}\right)^{\frac{N}{2}}e^{-\frac{1}{2\sigma_n^2}\|\mathbf{d}-\mathbf{Gm}\|_2^2}, \tag{14}$$

where N is the length of the data.