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The residual phase estimation of a seismic wavelet using a Rényi divergence-based criterion

Zhiguo Wang, Bing Zhang, Jinghuai Gao *

National Engineering Laboratory for Offshore Oil Exploration, Institute of Wave and Information, Xi'an Jiaotong University, Xi'an 710049, China

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A B S T R A C T

The residual phase estimation of a wavelet is commonly required in a stacked seismic section. Due to its ability to measure the non-Gaussianity of a seismic trace, kurtosis maximization by a constant-phase rotation is the most popular statistical phase estimation method. However, kurtosis does not always produce an optimal phase in complex seismic data such as low dominant frequency data. To overcome this difficulty, we have constructed a new Rényi divergence-based criterion to measure the non-Gaussianity of a seismic trace with a low dominant frequency. Through a numerical test using a Ricker wavelet with fixed frequencies and varying phases, this adjustable criterion is shown to be more sensitive than kurtosis to a change of wavelet phase. The robustness of this proposed method is demonstrated with two hundred different phase estimations of synthetic records with dominant frequencies of 20–50 Hz. Furthermore, very promising results have been obtained by applying this method to three data sets with a dominant frequency of 25 Hz in tight gas reservoirs at a depth of more than 4000 m.

1. Introduction

The control of the wavelet phase plays an important role in current seismic acquisition and processing (Trantham, 1994). For example, the phase of a seismic wavelet directly influences the results of deconvolution and inversion (Berkhout, 1977; Wiggins, 1978, 1985; Yua and Wang, 2011). In a stacked seismic section, each trace can be considered as a reflectivity series convolved with a zero phase wavelet (Levy and Oldenburg, 1987). Here, the desired zero phase of a wavelet is an accepted assumption to control the quality of processing and interpretation. But in real seismic data, despite the best efforts to control the phase of a wavelet during processing, the zero phase assumption is sometimes violated (van der Baan, 2008). Phase distortions arise due to a variety of reasons such as dispersion, attenuation, and band-limit (Levy and Oldenburg, 1987). Therefore, a further residual phase correction of the wavelet will be required in the stacked seismic section (Edgar and van der Baan, 2011; Gao and Zhang, 2010; Levy and Oldenburg, 1987; Longbottom et al., 1988; Lu, 2005; van der Baan and Fomel, 2009; Yu et al, 2012).

In general, there are deterministic and statistical ways to deal with residual phase estimation (Tygel and Bleistein, 2000). The deterministic method is to use existing well logs. A future phase correction is applied to the data such that they match the zero phase synthetics created from well logs (van der Baan, 2008). Unfortunately, the wavelet phase may vary laterally away from wells and vertically with time where well logs are not accessible (Xu et al., 2012). Thus, sometimes the deterministic method is limited, and an alternative statistical method is necessary. In contrast, statistical phase estimation does not require well logs and can estimate wavelet merely from seismic data. In this paper, we focus on statistical phase estimation.

Levy and Oldenburg (1987), Longbottom et al. (1988), and White (1988) first simplified the statistical phase estimation problem by assuming that a seismic wavelet can be described accurately by a constant-phase approximation for stationary data. For a constant-phase rotation the optimum phase renders the data maximally non-Gaussian, because the data with zero phase are maximally non-Gaussian compared with those of non-zero phase (van der Baan, 2008). Obviously, how to construct a measure of the non-Gaussianity of the data is the key for a statistical method. During the past three decades, many researchers have studied several criteria such as the kurtosis criterion (Wiggins, 1978), the parsimony criterion (Claerbout, 1977), the exponential transform criterion (Ooe and Ulrych, 1979), the Sech criterion (Sacchi, 2002), the Cauchy criterion (Sacchi, 2002), and the second- and third-order moments (Lu, 2005) to describe the non-Gaussianity of data. In most cases, the kurtosis criterion is the most popular choice. In a kurtosis-based algorithm, a series of constant-phase rotations is applied to the recorded seismic trace. The angle corresponding to the maximum kurtosis value determines the most likely wavelet phase (van der Baan, 2008). Unfortunately, real seismic data are usually nonstationary. To address this challenge, van der Baan (2008) developed a time-varying wavelet estimation method

* Corresponding author at: Institute of Wave and Information, Xi'an Jiaotong University, Xi'an 710049, China. Tel.: +86 29 82665060.
E-mail address: jhgao@mail.xjtu.edu.cn (J. Gao).

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to handle nonstationary nonminimum-phase wavelets. His method invoked a piecewise-stationarity assumption within moving analysis windows using kurtosis maximization by constant-phase rotation. As kurtosis is a higher-order statistics, the reliability of its estimation depends on the amount of data available. Accordingly, the chosen window requires a minimum length. Therefore, van der Baan and Fomel (2009) demonstrated a space-and-time-varying wavelet estimation method using regularized local kurtosis maximization, which makes van der Baan's method more robust with respect to smaller window sizes or regularization lengths. Analogous to local kurtosis, Fomel and van der Baan (2010, 2014) proposed local similarity with the envelope and local skewness as phase measures, which show a higher dynamic range and better stability in synthetic and field data examples.

However, the kurtosis maximization method does not always show a positive performance due to variable factors of seismic data. Based on synthetic and real seismic data, Xu et al. (2012) discussed some of the factors (e.g. reflectivity character, seismic frequency bandwidth, signal-to-noise ratio, and geology) that potentially affect the final phase estimated by a kurtosis-based method. For example, they indicated that there is a possible dominant frequency threshold for phase estimation using the kurtosis maximization method. Recently, we also observed that sometimes the kurtosis-based method is not stable enough to work with seismic data having a dominant frequency of 25 Hz in more than 4000 m deep tight gas reservoir, Ordos Basin, China. This illustrates that low dominant frequency data with a zero phase may not show more non-Gaussianity measured by kurtosis than those of a non-zero phase. Furthermore, Painter et al. (1995) show experimental histograms of reflection coefficients from 14 wells in Australia accurately approximated by symmetric Levy-stable probability density functions that do not have higher-order statistics, such as kurtosis. Thus, a more suitable criterion of non-Gaussianity in statistical phase estimates may be available for low dominant frequency data in deep reservoirs.

In information theory, there are many measures of non-Gaussianity of a distribution. Specifically, kurtosis is just an approximate measure, which is derived from negentropy based on the Kullback–Liebler divergence. Since 1967, Csiszár introduced and studied f-divergence including the Kullback–Liebler divergence, the Hellinger distance, and the total variation distance, which measures the difference between two probability distributions (Ali and Silvey, 1966). Based on f-divergence, it is convenient to yield a generalized family of non-Gaussian measurement.

In this paper, a new criterion based on the Rényi divergence (one of the Csiszár f-divergences) is constructed to indicate the non-Gaussianity of seismic data. This criterion is maximized when the locally observed phase is close to zero. Advantages of the new criterion include its higher sensitivity and better stability, which make it suitable for choosing phase corrections in low dominant frequency data in deep reservoirs. Using synthetic and three field-data examples, we demonstrate the properties and applications of the proposed criterion.

2. Method

The optimum phase could be estimated by applying a series of constant-phase rotations to the analyzed seismic trace. The phase rotation angle φ for which the trace is maximally non-Gaussian corresponds to the desired phase correction (Levy and Oldenburg, 1987; Longbottom et al., 1988; van der Baan, 2008; van der Baan and Fomel, 2009; White, 1988). In the time domain, the rotated trace $y_{rot}(n)$ can be obtained from the original trace $y(n)$ by:

$$y_{rot}(n) = y(n)\cos\varphi + H[y(n)]\sin\varphi, \tag{1}$$

where $H[\cdot]$ denotes the Hilbert transform. Since a zero phase wavelet contains the majority of its energy in a relatively narrow time range, its kurtosis is larger than that of any non-zero phase wavelet. So, when the kurtosis of $y_{rot}(n)$ reaches its maximum, the related angle is considered as the most likely phase correction for $y(n)$. This method for estimating the phase of a seismic wavelet is called the constant-phase rotation method (Levy and Oldenburg, 1987).

The constant-phase rotation method is based on the theory of entropy. The operation of rotating the seismic trace increases its non-Gaussianity. Based on Kullback–Liebler divergence, negentropy is used to measure the non-Gaussianity of a random variable, which is defined as (Hyvarinen et al., 2001):

$$H_{neg}(X) = \int_{-\infty}^{\infty} f(x)\log\frac{f(x)}{g_\sigma(x)}dx = H(X_{Gauss}) - H(X) \tag{2}$$

where $H(X)$ denotes the entropy of a continuous random variable X whose probability density function (PDF) is $f(x)$, X_{Gauss} is a Gaussian variable with the same variance as that of X. σ is the standard deviation of the data whose PDF is $g_\sigma(x)$ with a zero mean. Negentropy is non-negative, and will become larger as a random variable departs from Gaussianity. In practice, the negentropy of a random variable with a zero mean and unit variance can approximately be obtained as follows (Hyvarinen et al., 2001):

$$H_{neg}(X) \approx \frac{1}{12}E\left(X^3\right)^2 + \frac{1}{48}kurt^2(X), \tag{3}$$

where

$$kurt(X) = E\left(X^4\right) - 3 \tag{4}$$

is defined as the kurtosis of X. If X satisfies a symmetrical probability distribution function, the equation above reduces to:

$$H_{neg}(X) \approx \frac{1}{48}kurt^2(X). \tag{5}$$

Hence, the constant-phase rotation method is related to the minimum entropy deconvolution proposed by Wiggins (1978). To guarantee scale-invariance, Wiggins adopted the standardized fourth cumulant as a criterion to performing optimization, for a random variable with a zero mean, which is defined as:

$$kurt(X) = \frac{E\left(X^4\right)}{E^2\left(X^2\right)} - 3. \tag{6}$$

The kurtosis is usually calculated by Eq. (6) for the constant-phase rotation method. From Eq. (1) to Eq. (6), the derivation of a kurtosis-based wavelet estimation is clearly shown. Following the above idea of maximizing kurtosis, we attempt to construct a more suitable and general criterion instead of kurtosis.

With respect to Csiszár's divergences, the Rényi measure is considered in this study, which is given by (Rényi, 1960):

$$D^R(f,g) = \frac{1}{\alpha-1}\log\int f^\alpha(x)g^{1-\alpha}(x)dx, \qquad \alpha > 0, \alpha \neq 1, \tag{7}$$

where $f(x)$ and g are two probability density functions (PDF). For $\alpha = 1$, the Rényi divergence reduces to the classical Kullback–Liebler divergence (Hyvarinen et al., 2001). The Rényi divergence is one of the information measures that show a difference between two distributions, which is nonnegative and equals zero if and only if $f(x) = g(x)$.

If we use $f(x)$ to denote the PDF of the seismic data, and let g be a PDF of the normal distribution with the same variance as that of $f(x)$, then the Rényi divergence will become larger as the data departs from the