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An improved regularized downward continuation of potential field data

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A B S T R A C T

Downward continuation of potential field data plays an important role in interpretation of gravity and magnetic data. For its inherent instability, many methods have been presented to downward continue stably and precisely. In this manuscript, we propose an improved regularization operator for downward continuation of potential field data. First, we simply define a special wavenumber named the cutoff wavenumber to divide the potential field spectrum into the signal part and the noise part based on the radially averaged power spectrum of potential field data. Next, we use the conventional downward continuation operator to downward continue the signal and the Tikhonov regularization operator to suppress the noise. Moreover, the parameters of the improved operator are defined by the cutoff wavenumber which has an obvious physical significance. The improved operator can not only eliminate the influence of the high-wavenumber noise but also avoid the attenuation of the signal. Experiments through synthetic gravity and real aeromagnetic data show that the downward continuation precision of the proposed operator is higher than the Tikhonov regularization operator.

1. Introduction

The analytical continuation of potential fields is recognized as being a powerful tool in the transformation of geophysical potential fields (mainly in gravity and geomagnetic fields). Continuation of the potential field data above the level of measurement is known as upward continuation and in the opposite direction (below the level of measurement but must be above sources), it is known as downward continuation. During the potential field data processing and interpretation, upward continuation is often used to enhance the regional components in the original data by attenuating shallow surface sources manifestation and downward continuation is often used to enhance the detection of shallower sources by extracting the local anomalies and calculate the depth of the important shallowest sources.

Since the upward continuation is stable, its inverse operation, downward continuation, on the other hand, sharpens the geophysical anomalies in potential fields and results in instability at different levels of continuation. Many approaches have been proposed to solve this problem. These approaches can be classified into two categories: the directed method and the iterative method. The directed methods include the filter windows (Ku et al., 1971), the Wiener filter theory (Clarke, 1969; Pawlowski, 1995; Trompat et al., 2003), the boundary element method (Xu, 2001), the multiscale edge theory (Trompat et al., 2003), the integrated horizontal derivative and compensation approaches (Cooper, 2004) and the Tikhonov regularization theory (Abedi et al., 2013; Ferguson et al., 1988; Li et al., 2013; Pašteka et al., 2012). The iterative methods include Taylor series approximation method (Fedi and Florio, 2002; Peters, 1949; Zhang et al., 2013), techniques based on the equivalent-layer theory (Dampney, 1969; Hansen and Miyazaki, 1984; Leão and Silva, 1989; Li and Oldenburg, 2010; Oliveira et al., 2013), the spline function method (Wang, 2006), the iterative Fourier equivalent source method (Xia et al., 1993) as well as some other iterative methods (Dmitriev and Dmitrieva, 2012; Guspi, 1987; Ma et al., 2013; Strakhov and Devitsyn, 1965; Xu et al., 2007; Zeng et al., 2013).

According to the comparative analysis made by (Zeng et al., 2013) and (Zhang et al., 2013), the performance of Tikhonov regularization is better than almost all of iterative methods which mentioned in previous literatures. In addition, many iterative methods have some of their inherent disadvantages. For example, the semiconvergence property (the downward continuation error of these iterative methods is decreasing first and increasing afterwards) results in the iterative numbers and stopping criterions of these iterative methods are not easy to determine. Of course, the Tikhonov regularization method also has some difficulties in practice, such as the saturation effect (it means that the order of error estimate between the solution of Tikhonov regularization method and the exact solution cannot be infinitely improved along with the improvement of the smoothness properties (Engl et al., 1996)) and the choice of the regularization parameter. In this study, we present an improved regularization operator for downward continuation based on the characterization of the potential field spectrum. First of all, an ad hoc wavenumber named the cutoff wavenumber is defined to divide the potential field spectrum and determine the regularization parameters. Then, we apply the conventional downward continuation operator and the Tikhonov regularization operator to downward continue the two parts of potential field spectrum, respectively. We compare the downward

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continuation precision of the improved operator with the Tikhonov regularization operator based on synthetic gravity and real aeromagnetic data. The results show that the method for the choice of the regularization parameters is effective and our proposed operator attains better downward continuation accuracy than the Tikhonov regularization operator.

2. Regularization for downward continuation of potential field

The analytical continuation is defined as the well-known Dirichlet integral, which is also known as Fredholm integral equation of the first kind (Blakely, 1996):

$$u(x, y, -h) = \frac{h}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{u(\xi, \eta, 0)}{\left[(x-\xi)^2 + (y-\eta)^2 + h^2\right]^{3/2}}d\xi d\eta, \qquad (1)$$

where $u(x,y,0)$ and $u(x,y,-h)$ are respectively the potential field data at a lower observation level and at a vertical distance of h above. Applying 2D Fourier transform to Eq. (1) yields:

$$U\left(\omega_x, \omega_y, -h\right) = e^{-h\omega_r} \cdot U\left(\omega_x, \omega_y, 0\right), \qquad (2)$$

where $U(\omega_x,\omega_y,-h)$ and $U(\omega_x,\omega_y,0)$ denote the Fourier transform of $u(x, y,-h)$ and $u(x,y,0)$, respectively; ω_x and ω_y are the wavenumbers in the x- and y-directions and $\omega_r = \sqrt{\omega_x^2 + \omega_y^2}$ is the radial wavenumber. $e^{-h\omega r}$ is the upward continuation operator and vice versa for the downward continuation operator. It has been shown that for the downward continuation operator, high wavenumbers have high amplitudes, and hence any noise in the data at high wavenumbers is enhanced, rendering the operation numerically unstable. This can be achieved by formulating the process as an inverse problem and deriving a regularization operator. The Tikhonov regularization operator has the following form (Abedi et al., 2013; Li et al., 2013; Zeng et al., 2013):

$$R = \frac{1}{1 + \alpha e^{2h\omega_r}} \cdot e^{h\omega_r}, \qquad (3)$$

where α is the regularization parameter. It is clear that the regularization operator consists of two parts: the classical downward continuation operator $e^{h\omega r}$ and the regularization low-pass filter $\frac{1}{1+\alpha e^{2h\omega_r}}$. It is clear that the regularization low-pass filter attenuates the influence of the high-wavenumber components and ensures the stability of the regularization operator. For simplicity, as we know, the observed potential field can often be divided into the deep-source long-wavelength potential field and the shallow-source short-wavelength potential field. The effects of deeper-source dominate the low-wavenumber part of the spectrum, whereas shallower source and white noise dominate the higher wavenumbers. In this study, the potential field effect of deeper sources is regarded as the signal. Other parts of the spectrum representing shallow sources or fluctuations due to measurement error are considered undesirable components of the overall potential field and are assumed to be noise. Naturally, if we want to design a better regularization low-pass filter, it is desired to all-pass the signal parts while at the same time suppressing the noise parts. As a result, we proposed the following improved regularization low-pass filter:

$$\text{filter} = \begin{cases} 1 & \text{if}\,\omega_r \le \omega_c \\ \dfrac{1}{1 + \alpha e^{2h\omega r}} & \text{if}\,\omega_r > \omega_c \end{cases} \qquad (4)$$

where ω_c is the cutoff wavenumber which divides the potential field spectrum into two parts.

3. Choice of the filter parameter based on the characterization of the potential field spectrum

The key of the regularization low-pass filter in Eq. (4) is the choice of the filter parameter α. In previous work, Pašteka et al. (2012) suggest the use of the C-norm criterion which is very close to the concept of the L-curve criterion to optimally define the value of the regularization parameter. Li et al. (2013) and Zeng et al. (2013) used the L-curve criterion for choosing the optimal regularization parameter. The procedures of these two methods for the choice of the regularization parameter can be summarized as follows. First, used values of the regularization parameter α are changed in a geometrical sequence (with the common ratio greater than 1), starting with very small values and finishing with relatively large ones. Then, after repeatedly performing the downward continuations with various α, they get optimal regularization parameters by some characters of the specific curves respectively. For example, the local minimum for the C-norm curve, the corner point or the maximum curvature point for L-curve. From the abovementioned discussion, we know that the calculation of the optimal regularization parameters by L-curve and C-norm is a time-consuming process. In this manuscript, the optimal selection of the filter parameters is achieved by consideration of the spectral characteristics of potential field data.

Spector and Grant (1970) illuminated that a potential field spectrum has a characteristic shape which is dominated by the effect of source depth. Fig. 1 shows an assumptive example for the radially averaged power spectrum of potential field. Firstly, deeper sources have correspondingly steeper spectral slopes. Observational errors, if statistically uncorrelated, are represented by white noise, the mean spectral value of which is a constant. Noise dominates the potential field spectrum at higher wavenumbers. Secondly, since the spectrum has a sloping line representing the signal and a flat line representing noise, a least-squares, linear spline approximation to the spectrum seems appropriate. The signal part of the spectrum is fit separately from the noise. At last, the noise is constrained to have zero slope, and both solutions are constrained to merge at the wavenumber ω_c'. Slope and intercept of the signal and mean noise level are fit by least squares to spectral samples partitioned at ω_c'.

In order to regularize the downward continuation function without overfiltering or underfiltering the data, it is necessary to choose the regularization parameters, α and ω_c, which taper the continuation function beginning near the wavenumber ω_c'. Therefore, the regularization low-pass filter can retain the maximum amount of signal with a minimum

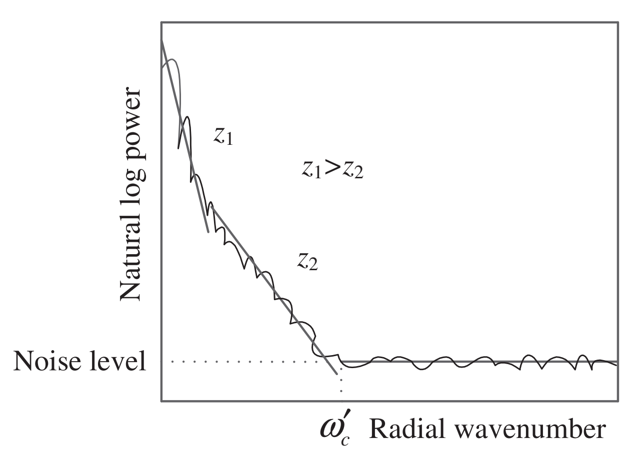

Fig. 1. The potential field spectrum for sources at two different depths, z_1 and z_2. Slopes of line segments are proportional to depth with steeper slope indicating greater depth. White noise contributes the flat part of the spectrum. The wavenumber ω_c' separates signal from noise. This is the assumption used in the regularization process.