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Displacement currents in geoelectromagnetic problems

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ABSTRACT

The influence of displacement currents in conventional geoelectromagnetic (GEM) methods using unimodal transversal electric (TE) or multimodal TE and TM (transversal magnetic) fields is only significant at very high frequencies in the frequency domain or at extremely early times in the time domain. The transient process in the latter includes three stages: the propagation through air, the propagation through earth and the diffusion within the earth. The influence of displacement currents is significant mainly during the former two stages, normally up to several tens to a few hundreds of nanoseconds. The behavior is essentially different in novel GEM methods using a vertical electric dipole (VED) or circular electric dipole (CED) sources of unimodal TM-fields. Under certain geoelectric conditions, the influence of displacement currents in these methods might be crucial at late times as well. This happens, if the model consists of insulating layers. In the absence of displacement currents, such layers would totally mask underlying structures. However, TM-fields including displacement currents depend on geoelectric parameters below insulating layers at late times.

1. Introduction

In most cases, displacement currents (DSPC) are neglected when electromagnetic fields are applied for solving real geoexploration problems. Only in very high resistivity environments, such as permafrost, magmatic rocks, dry sands, carbonates, etc., the influence of DSPC might be significant at frequencies greater than roughly 100 kHz (Sinha, 1977). Such frequencies are only used in a very few existing methods such as RMT (radiomagnetotellurics). The situation in the time domain is even more severe since DSPC are significant at extremely early times in the order of nanoseconds, which are far beyond the working range of all existing transient electromagnetic (TEM) instruments. To the best of our knowledge, such a range has been realized in a single experimental TEM system developed in the framework of the Very Early Transient Electromagnetic (VETEM) project (Wright et al., 1996). However, the use of this system has never been moved beyond the experimental stage.

There is only one working geophysical method, in which DSPC play a crucial role. It is GPR (ground penetrating radar), which operates at frequencies greater than 25 MHz (e.g. Smith and Jol, 1995). At such frequencies, electromagnetic energy travels as waves in most environments and the measured signals are treated similarly to those in seismic methods. Therefore, GPR is not in the realm of inductive EM and as such it falls beyond the scope of this study.

Due to the lack to non-existence of practical applications in time domain EM, detailed theoretical investigations of DSPC based on numerical solutions for complicated 2D/3D models were mostly limited by frequency domain methods (e.g. Kalscheuer et al., 2008). There are very few studies in the time domain, which investigated the influence of DSPC on transient response based on numerical and/or analytical solutions and all them were carried out for relatively simple models (Bhattacharyya, 1959; Goldman et al., 1996; Wait, 1982; Weidelt, 2000).

To the best of our knowledge, all existing studies of DSPC both in frequency and time domains were carried out either for unimodal TE (transversal electric) fields or for bimodal TE–TM (transversal magnetic) fields. In recent years, there was an increasing interest in the use of unimodal TM-fields, which are highly sensitive to thin resistive structures and to lateral resistivity variations (e.g. Goldman and Mogilatov, 1978; Holten et al., 2009; Mogilatov and Balashov, 1996).

This study of DSPC includes both unimodal TE-fields generated by a vertical magnetic dipole (VMD) and unimodal TM-fields generated by a circular electric dipole (CED) on the surface of selected 1-D models. Note that a CED represents a surface analog of a more conventional vertical electric dipole (VED) source embedded within the earth (Mogilatov, 1992). The study led to unexpected results: contrary to unimodal TE-fields and multimodal TE–TM fields, which are only affected by DSPC at very shallow depths (early times), the influence of DSPC in unimodal

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TM-fields might be significant and even crucial at large depths (late times) as well.

2. VMD (TE-field), early times (high frequencies)

Let's consider a model consisting of a boundary ($z = 0$) between two homogeneous half spaces with arbitrary resistivities and dielectric permittivities. The magnetic permeability in both half spaces equals that in vacuum. A VMD having moment M_z is located in the upper half space at point $z = z_0$ of a cylindrical coordinate system with the z-axes pointed up (Fig. 1a). At time instant $t = 0$, the moment abruptly drops to zero. The analytical solution in frequency domain is well known (e.g. Wait, 1982). In fact, it is identical to that in the quasi-static approximation with the only difference in the expressions for wave numbers:

$$k_i^2 = -\mathbf{i}\omega\mu_0/\rho_i - \omega^2\mu_0\varepsilon_i, \tag{1}$$

$$k_i^2 = -\mathbf{i}\omega\mu_0/\rho_i (i = 0, 1). \tag{2}$$

Here Eq. (1) represents wave numbers including DSPC and Eq. (2) represents wave numbers in the quasi-static limit. Such an insignificant difference in the frequency domain makes it extremely difficult up to impossible to perform a numerical Fourier transform into the time domain. Fortunately, the solution in time domain for the model under consideration, can be obtained analytically using the Laplace transform by substituting $\mathbf{i}\omega = \gamma_i - s$, $\gamma_i = 1/(2\rho_i\varepsilon_i)$, $i = 0, 1$.

Let's consider the practically interesting case, when VMD is located on the surface of a homogeneous earth ($z_0 = 0$). This problem has been considered by Bhattacharyya (1959). However, our solution considerably differs from that of Bhattacharyya (1959) by the appearance of a leading edge with infinitely large amplitude (point source, ideal step-off excitation). By applying the Laplace transform to the full frequency domain solution, one obtains the following equations for electrical and vertical magnetic fields, E_ϕ and dB_z/dt, respectively (Goldman et al., 1996):

$$E_\phi = \frac{M_z\mu_0}{2\pi}(A_1 - A_0), \tag{3}$$

where

$$A_i = \frac{\overline{\rho}}{\mu_0}\cdot\frac{T_i^2}{r^4}\cdot\left[\int_{-\infty}^{\infty}\left(I_i^{(2)}\cdot T_i - I_i^{(1)}\right)\cdot U(\tau-T)\cdot\varphi_i(\tau)\cdot U(t-\tau)\cdot d\tau + \right.$$
$$+\varphi_i(T_i)\cdot\left(1+\frac{\gamma_i^2T_i^2}{2}\right)\cdot U(t-T_i) - \varphi'_{i\tau}(T_i)\cdot T_i\cdot U(t-T_i) + \tag{4}$$
$$\left. +\varphi_i(T_i)\cdot T_i\cdot\delta(t-T_i)\right].$$

$U(x)$ and $\delta(x)$ are the Heaviside step-off function and the Dirac delta function, respectively, $I^{(n)}$ is the n-th derivative of $I_i \equiv I_i^{(0)} = I_0\left(\gamma_i\sqrt{\tau^2 - T_i^2}\right)$ with respect to T, I_0 is the modified Bessel function of zero order, $T_i = r/c_i$ is the first arrival time, $c = 1/\sqrt{\mu_0\varepsilon_i}$ is the light velocity in the i-th medium,

$$\varphi_i(\tau) = -\frac{\exp(-\gamma_i\tau)}{\tau^2}\cdot\{1 + \gamma_i\tau + \exp[-2\overline{\gamma}(t-\tau)]\cdot(2\overline{\gamma}\tau - \gamma_i\tau - 1)\},$$
$$\overline{\gamma} = 1/(2\overline{\rho}\overline{\varepsilon}), \overline{\varepsilon} = \varepsilon_1 - \varepsilon_0, \overline{\rho} = \rho_0\rho_1/(\rho_0 - \rho_1), i = 0, 1.$$

These equations are suitable for calculations. The equations are further simplified by considering the practically important case, when the upper half space is air ($\rho_0 = \infty$). Then $\gamma_0 = 0$ and the expression for A_0 in Eq. (4) becomes:

$$A_0 = \frac{\overline{\rho}}{\mu_0}\cdot\frac{T_0^2}{r^4}\left[\phi_0(T_0)U(t-T_0) - \varphi'_{0\tau}(T_0)T_0U(t-T_0) + \varphi_0(T_0)T_0\delta(t-T_0)\right]. \tag{5}$$

The expression for the time derivative of the vertical magnetic field is obtained from Eq. (3) by simple differentiation:

$$\dot{B}_z = -\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\cdot E_\varphi. \tag{6}$$

Fig. 2 shows the transient responses of E_ϕ for different relative dielectric permittivities $\varepsilon_1/\varepsilon_0$ of the earth. It is important to emphasize that the responses are calculated for the ideal point source and step

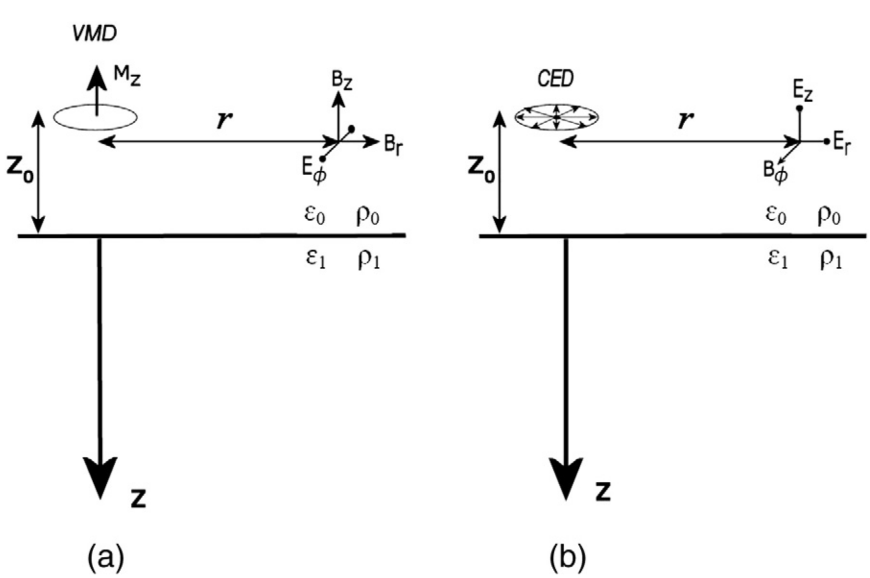

Fig. 1. Model geometry.