Journal of Applied Geophysics 105 (2014) 169–179

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Two-dimensional geomagnetic forward modeling using adaptive finite element method and investigation of the topographic effect

Mehdi Shahmirzae Jeshvaghani [a], Mehrdad Darijani [b,*]

[a] Institute of Geophysics, University of Tehran, Tehran, Iran
[b] Department of Earth Science, Memorial University of Newfoundland, St. John's, NL, A1B 3X5, Canada

ARTICLE INFO

Article history:
Received 23 October 2013
Accepted 20 March 2014
Available online 28 March 2014

Keywords:
Finite element
Forward modeling
Geomagnetic
Topography
Triangular grid
2-D

ABSTRACT

Forward modeling approach is a major concept in geophysical exploration and also a key factor in the development of inversion algorithms. Finite element method for two-dimensional (2-D) geomagnetic forward modeling is based on numerical solution of the Laplace equation. In this paper we present a fast and accurate adaptive finite element algorithm for forward modeling of 2-D geomagnetic structures. Our method is stable and is reliable to recover 2-D magnetization distribution with complex shapes. It uses an unstructured triangular grid which allows modeling the complex geometry with the presence of topography. The Galerkin's method is used to derive the systems of equations. Then, the conjugate gradient solver with incomplete LU decomposition as the preconditioner is used to solve the system of equations. To ensure numerical accuracy, iterative mesh refinement is guided by a posteriori error estimator. We validate our algorithm in simple geometry by analytical technique. The tests on synthetic data illustrate a good performance of the method in mapping the complex geometry of the magnetic sources with topography. The magnetic responses of the model have proved to be different in the presence of topography. Therefore, it is highly recommended to consider the effects of topography on interpretation. Finally, we applied numerical FEM algorithm to real data set providing fine recovery model of the shallow high mineralized crustal setting of Soltanieh region, Iran.

1. Introduction

Magnetic survey is one of the most popular geophysical techniques for fast mapping of large areas. The main goal of magnetic prospecting is to infer both the geometry and magnetization of the geologic structure that causes the observed magnetic anomalies. However, akin to other potential-field methods, interpretation of magnetic field anomalies is non-unique because more than one distribution of magnetization (i.e., magnetic dipole moment per unit volume) and source geometry can explain the same observed magnetic anomaly (García-Abdeslem, 2008). Although methods to model isolated magnetic anomalies using simple geometries such as spheres, cylinders, and prisms exist, the approximation of the source body with one or more layers of contiguous prisms is often justified (Bhattacharyya, 1980).

High-resolution magnetic surveys have advanced by allowing scattered geological observations to be integrated into regional interpretations (Gunn, 1997). As ground geophysical surveys became important tools for mineral exploration, new techniques allowed 2-D models of geological structures to be constrained using the results of magnetic fields along individual profiles (Nettleton, 1942). There is a plethora of modeling tools available for building 2-D and 3-D geological models by potential-field data (excellent collections of papers can be found in Caratori Tontini et al., 2009; Hamilton and Jones, 1992; Houlding, 1994; Jessell, 2001; Pflug and Harbaugh, 1992; Pignatelli et al., 2011; Turner, 1992). For example, Talwani (1965) approximated real sources by sets of stacked lamina for magnetic data and Plouff (1976) developed a closed-form equation for the gravity anomaly of a finite-thickness horizontal plate, which is useful for terrain corrections. Pignatelli et al. (2011) approximated the subsurface magnetization distribution by a set of prismatic cells with constant magnetization and Caratori Tontini et al. (2009, 2012) developed a 3-D forward modeling equations for the magnetic field based on a 3-D Fast Fourier Transform (FFT) of the magnetization distribution.

To investigate subsurface structure from potential data such as magnetic data, various methods have been developed. Blakely (1995) divided them into three categories: the forward modeling method, the inverse modeling method, and the data enhancement and display method. The forward method, based on geological and geophysical intuition, constructs an initial model for the source body and then computes the model's magnetic effect that is to be compared with the observed anomaly (Shin et al., 2006).

Forward modeling of geophysical potential fields plays an important role in direct interpretation of magnetic anomalies. Geological structures such as faults, folds, and veins are often complex and geophysical

* Corresponding author. Tel.: +1 709 771 6757.
E-mail addresses: m.shahmirzae@ut.ac.ir (M.S. Jeshvaghani), m.darijani@mun.ca (M. Darijani).

http://dx.doi.org/10.1016/j.jappgeo.2014.03.016
0926-9851/

parameters such as intensity of magnetization are usually also heterogeneous. Consequently, 2-D or 2.5-D potential field modeling has certain limitations. Barnett (1976) and Okabe (1979) developed forward modeling techniques for gravity and magnetic anomalies due to homogeneous polyhedral bodies (Pohanka, 1988; Wang et al., 1980). It is easy to approximate a homogeneous polyhedral body composed of a set of polygonal facets of the body to the real arbitrary shape of a geological body, and the degree of approach depends on the number of polygonal facets and the selection of vertexes. Moreover, an inhomogeneous body can be divided into several smaller homogenous ones. Therefore, the calculation of gravity and magnetic anomalies due to a homogeneous polyhedral body is more practicable and more significant than that from other types of bodies, especially for high resolution gravity and magnetic survey (Yao and Changli, 2007). With rapid advancement in computation facilities, software technology, and numerical methods in applied mathematics, solvability of geophysical forward problems has increased immensely (Roy, 2008). Since the subsurface of the earth has a complex geometry, solution of any realistic inverse problem demands solution of the forward problems for similar type of subsurface structure. Thus, numerical methods entered with all its well-known tools, finite difference, finite element, integral equation, volume integral, boundary integral, and hybrids method (Roy, 2008).

The Finite element method utilizes a variational problem that involves an integral of the differential equation over the model domain. The variational integral is evaluated as a sum of contributions from each finite element. The result is a set of algebraic equations of the approximate solution. This system of equation has a finite size rather than the original infinite-dimensional partial differential equations (Ismail-Zadeh and Tackley, 2010). Among the advantages of this approach in modeling complex irregular regions are the use of non-uniform meshes to reflect solution gradations, the treatment of boundary conditions involving fluxes, and the construction of high-order approximations. Estimates of discretization errors may be obtained for reasonable costs. These may be used to verify the accuracy of the computation, and also to control an adaptive process whereby meshes are automatically refined to compute solutions to desired accuracies in an optimal fashion (Babuska and Rheinboldt, 1979; Babuska et al., 1983, 1986; Bern et al., 1999 for more details).

In this paper, we present the 2-D geomagnetic forward problem using adaptive FEM procedure. Our algorithm implements an unstructured triangular mesh, which allows us to model complex structures with topography and irregular magnetic body even when the ground surface has the magnetic susceptibility.

2. Methodology

The goal of this section is to introduce a finite element approach to solve geomagnetic problems in terms of a scalar potential in two dimensional bounded domains. Several numerical methods have been developed to solve magnetostatic problems, but numerical results show that the scalar potential approach is especially efficient. For this purpose, the classical magnetostatic model is obtained by neglecting the time derivatives in Maxwell's equation and the given divergence — free stationary source current density j, the magnetic field $\vec{H}$ and the magnetic induction $\vec{B}$ satisfy the following equations (Bermúdez et al., 2008):

$$\nabla \times \vec{H} = j \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

The fields $\vec{H}$ and $\vec{B}$ are linked by constitutive relation:

$$\vec{B} = \mu \vec{H} \tag{3}$$

where μ is the magnetic permeability.

In the region where it is free of the source of magnetic field, there is no electric current or displacement current. For this region, it is introduced a scalar potential $u$which defines the magnetic field:

$$\vec{H} = -\nabla u. \tag{4}$$

With Eq. (2) and Eq. (3) we have:

$$\nabla \cdot \vec{B} = \nabla \cdot (-\mu \nabla u) = 0 \tag{5}$$

which is a PDE of Laplace type.

At interface of regions with different permeability, the normal component of $\vec{B}$and tangential component of $\vec{H}$ must be continuous. The potential u must be continuous for this interface (Simkin and Trowbridge, 1979). To pose the problem in the bounded domain, Ω we have to add adequate boundary condition to Eq. (5) (Bermúdez et al., 2008):

$$\vec{B} \cdot \vec{n} = g_N \text{ on } \partial\Omega.$$

g_N is a given data function and $\vec{n}$ is the outward normal vector to $\partial\Omega$ (Fig. 1). By virtue of Eq. (2) the data g_N must have zero mean (compatibility condition) (Bermúdez et al., 2008):

$$\int g_N = 0.$$

For geomagnetic modeling and simulation of the earth core field, we specify normal component of induction magnetic field using ambient geomagnetic field, B_0:

$$\vec{n} \cdot \vec{B} = \vec{n} \cdot \mu \nabla u = \vec{n} \cdot \vec{B}_0 = g_N. \tag{6}$$

This normal component field is specified under global definition of ambient magnetic field.

For finite element solution of Eq. (5) with specific boundary condition, we use weak formulation because classical solution of boundary value problem may not be available. To derive the weak form of Neumann problem for any test function,v is defined on Ω. Multiplying by Eq. (5) and integrating over model Ω, then performing the integration calculation using Green's formula and substitute the boundary condition yields (Gockenbach, 2006):

$$\int_\Omega \nabla \cdot (-\mu \nabla u) v d\Omega = 0 \qquad v \in H^1(\Omega)$$

$$\int_\Omega \mu \nabla u \cdot \nabla v - \int_{\partial\Omega} v(\mu\, n \cdot \nabla u) = 0$$

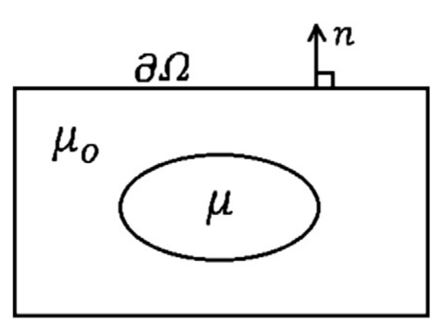

Fig. 1. Magnetic material in domain Ω with the outer Neumann boundary ∂Ω, and the outward unit normal vector ***n*** on ∂Ω.