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Body-growth inversion of magnetic data with the use of non-rectangular grid

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ABSTRACT

Recently, inversion of magnetic data to recover a distribution of magnetic susceptibility has been widely used for mineral exploration and other problems. However, the commonly used grid-based techniques have some practical difficulties, namely, degraded resolution with depth, increased computational cost with the size of the problem, and the influence of regional field. Especially, most of inversion techniques employ rectangular grid for division, which is inconvenient to represent complicated magnetic structure and actual topography.

We presented the magnetic version of body-growth inversion method with the use of non-rectangular grid. Essentially, this method is a new implementation scheme of 2-D and 3-D magnetic inversion, inherited from the gravity inversion by means of growing bodies, previously developed. For simple modification, we adopt non-rectangular grid to divide the subsurface region into a set of isoparametric finite elements rather than the commonly used rectangular grid of prismatic cells. This allows a better representation of actual topography and complicated magnetic structure in forward modeling. Additionally, the calculations of magnetic field and sensitivity matrix are implemented by the Gauss–Legendre quadrature rather than analytic formulae.

We tested the method by using synthetic data for two 2-D and three 3-D models and applied it to field data. Resultantly, we conclude that the method has some advantages such as: better representation of actual topography, automatic imposing of the positive magnetic susceptibility, and possibility to separate the regional field and remove its effect to inversion result. Thus, it seems to be a better alternative to the traditional grid-based techniques when inverting local bodies with complicated shapes under mountainous terrain.

1. Introduction

Probably, magnetic prospecting is the one of the geophysical exploration methods with the oldest history and widest spectrum of application, such as exploration of ore deposits, petroleum, groundwater, environmental, and satellite survey. The rapid advancements in hardware for magnetic observation during the last few decades have triggered an avalanche of new developments in data processing, forward modeling, and inversion, which in turn led to new challenges in instrumentation, and again accelerate multidisciplinary combination of various techniques (Nabighian et al., 2005a).

Many new ideas have been proposed and implemented in the magnetic inversion. Recently, most of these methods are based on the grid-based magnetic inversion (briefly, magnetic imaging), in which the geometric parameters of the model are fixed by a discretizing grid; the magnetic susceptibilities of each cell should be inverted (Li and Oldenburg, 1996a; Pilkington, 1997; Portniaguine and Zhdanov, 2002). Although such grid-based inversion methods remarkably improved the interpretation of magnetic data under geologically complicated situations, they had many practical difficulties: the resolution power is decreased with depth in general, the computational cost increases rapidly with the size of the problem, the regional field affects the inversion result, etc.

To overcome such difficulties, Li and Oldenburg (1996a) used weighting functions that compensate the natural decay of the magnetic field with depth (or distance) to enhance the depth-resolution. Li and Oldenburg (2003) proposed a wavelet compression method to reduce the storage of sensitivity matrix and a logarithmic barrier method to ensure the positivity of recovered magnetic susceptibility. Also, Li and Oldenburg (1996b) proposed a method for separating regional and residual magnetic fields through a preliminary 3-D magnetic inversion.

Similarly, Portniaguine and Zhdanov (2002) proposed a compression technique to speed up the computations and decrease the memory size, and employed a minimum support stabilizing functional to recover a sharp, focused image of magnetic structure.

As many authors discussed, it is essential to incorporate a priori knowledge (e.g., surface geology, drilling, and petrophysics) in the inversion process as a type of constraints in magnetic imaging. However, the existence of so many attempts for magnetic inversion proves that there are still some unresolved problems in this area.

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On the other hand, a variety of semiautomatic methods (e.g. SPI, ELW), based on the use of derivatives of the magnetic field have been developed for the determination of magnetic source parameters such as locations of boundaries and depths (Salem et al., 2007).

Meanwhile, Camacho et al. (2000) proposed a new gravity inversion method for determining the shapes of anomalous bodies with pre-established density contrasts. They pointed that their proposal is related to the "bubbling" method of Zidarov (1990) and the "open-reject-fill" method of René (1986). However, as opposed to Zidarov's (1990), the method works step-by-step on a chosen discretization of the subsurface region, expanding the anomalous bodies, for which prismatic elements are selected by means of balanced minimization of data fitness and model smoothness. Camacho et al. (2002) presented a 3-D implementation code written in Fortran 77 of this method and applied it to simulated and real data. More recently, Camacho et al. (2007) and Schiavone and Loddo (2007) reported some methodological improvements and application examples of this method. As has been highlighted by Camacho et al.(2000), this method has some new features: (1) a 3-D context is supposed, (2) nongridded nonplanar inaccurate data are accepted, (3) a "maternal" structure is not required, (4) a simple regional trend can also be simultaneously determined, (5) matter expansion can appear everywhere (not only for contiguous elements), (6) not requiring "contiguous" expansion lets one consider nonregular subsoil partitions, (7) if previous qualified models exist, they can be incorporated, (8) positive and negative density contrasts are simultaneously accepted. In this paper, we call this method a body-growth inversion (BG inversion, briefly), for convenience.

Here, we note that employing non-rectangular grid, cell (element), and numerical quadrature is an effective manner to represent the complicated terrain, anomalous bodies, and fields in geophysical interpretation. For an example, García-Abdeslem and Mart'ın-Atienza (2001) evaluated the gravity terrain correction by combining analytic and numerical methods of integration. Also, J. García-Abdeslem (2008) developed a method for 3D forward modeling of the total-field magnetic anomaly caused by a layer with its top and bottom surfaces represented by 2D Gauss–Legendre functions by combining both analytic and numerical methods of integration. For the other examples, a computation method of regional gravity anomaly was developed based on element shape functions used in finite element analysis (Mallick and Sharma, 1999). Authors approximated a regional gravity by employing isoparametric elements of second-order eight nodes, or third order twelve nodes. All of the authors mentioned above, employed Gauss–Legendre quadrature technique for numerical integration.

Beyond the gravity and magnetic methods, that Loke and Barker (1995) derived the analytical formula of partial derivative for homogeneous earth in electrical resistivity method and evaluated them numerically using Gaussian quadrature for multiple integrals; Li and Pek (2008) presented an adaptive unstructured mesh finite element procedure for improving the quality of numerical solutions in the magnetotelluric modeling.

This study is motivated from the earlier studies discussed above and based upon our own work on the numerical forward modeling of gravity and magnetic fields by multiple Gauss–Legendre quadrature (Kim et al., 2009, see Appendices A and B). In this paper, we propose a magnetic version of body-growth inversion with the use of non-rectangular grids, composed of first-order iso-parametric finite elements. In other words, the magnetic inversion is implemented by adopting the principle of gravity BG inversion of Camacho et al. (2000) without remarkable modifications, while we calculated the theoretical magnetic anomaly and sensitivity matrix numerically by multiple Gauss–Legendre integration on isoparametric elements grid. Our method is tested by using synthetic data for two 2-D, three 3-D models and applied to interpret a real field data in central Korean peninsula.

2. Methodology

2.1. Body-growth procedure for magnetic data

The wide exchange of methodologies and techniques of data processing, modeling and inversion was a traditional trend in gravitational and magnetic exploration throughout last decades (Nabighian et al., 2005a,b), and is an important way of further developments. Many methods, originated from gravity prospecting, have been applied to magnetic prospecting, and vice versa. This is why both gravitational and magnetic fields are potential fields in the viewpoint of field theory. Hence, the body-growth inversion, established by Camacho et al. (2000) for gravity data, can be naturally employed for magnetic data.

2.1.1. 3-D case

Given a set of N-data of total-field magnetic anomaly $d_i = T(P_i)(i = 1, \cdots, N)$, we seek to construct a 3-D susceptibility distribution beneath the surface. In this paper, we suppose data points $P_i(x_i, y_i, z_i)(i = 1, \cdots, N)$ are located above the earth's surface with arbitrary topographic relief.

Let the subsurface region be discretized into a set of M-cells by a 3-D grid (rectangular or non-rectangular) and assume a constant magnetic susceptibility $m_j = \kappa_j(j = 1, \cdots, M)$ within each cell. Then, the total-field magnetic anomaly is related to the susceptibility distribution by a linear relationship

$$d_i = \sum_{j=1}^{M} A_{ij} m_j, i = 1, \cdots, N \quad (1)$$

where A_{ij} characterizes the contribution of an unit magnetic susceptibility source filled in the j-th cell, to the i-th datum, often called Fréchet derivative or Kernel, and A(N × M matrix) is called a sensitivity matrix, the calculation of which will be described in Section 2.2. At this situation, the problem is how to estimate the unknown model vector $\mathbf{m} = (m_1, \cdots, m_M)^T$ which satisfying Eq. (1) or its vectorial notation:

$$A\mathbf{m} = \mathbf{d} \quad (1')$$

from given data vector $\mathbf{d} = (d_1, \cdots, d_N)^T$. To do this, many authors minimized a objective functional composed of data misfit and model norm (Li and Oldenburg, 1996a; Portniaguine and Zhdanov, 2002); Chasseriau and Chouteau (2003) presented an inversion method based on the stochastic