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# An extension of gravity probability tomography imaging

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A R T I C L E I N F O

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Two categories of gravity inversion methods based on the classification of the inversion results are (i) direct inversion of the density contrast using a linear or nonlinear algorithm, and (ii) inversion of the source distribution in a purely probabilistic sense, in which the inversion results are equivalent physical parameters between $+1$ and $-1$ that represent the influence or deficit of density relative to the density of the host volume. The second of these methods is the easier and more stable of the two, but in many cases the density-contrast model is preferred for the recognition of particular lithologies. Also, the inversion processing method requires specific geological information constraints to be added, both to make the result more meaningful and to offset the inherent non-uniqueness of the inverted static potential field. The present study extends the scope of the probability tomography method by introducing an iterative procedure to directly invert the density. In the proposed method, the initial model is produced by multiplying a small density by its probability of occurrence, and then an iterative method is used to refresh the model until both forward data and observed data fall within a given error margin. A density range restriction for each subdivided rectangular cell is added at each iteration to improve the focusing effect. Tests of the proposed method using two simple one- and two-prism models showed that the inversion of gravity data yields meaningful geological results.



## 1. Introduction

Gravity inversion is an important tool for retrieving the model parameters from measured gravity data. It is widely used for mapping geological structures in tectonic studies, resource exploration and engineering investigations, especially at the reconnaissance stage of these applications.

Inversion methods have undergone considerable development in the past decades. One type of gravity inversion is the direct determination of the three-dimensional subsurface distribution of density contrasts and includes both linear and nonlinear methods. Li and Oldenburg (1998) proposed two such techniques. In the first of these, the gravity data is transformed into pseudomagnetic data using the Poisson's relation, and the inversion is carried out using a 3-D general magnetic inversion algorithm (Li and Oldenburg, 1996). In the second technique, the gravity data is inverted to recover a minimum structure model, and the final density distribution is obtained by minimizing a model objective function. Nonlinear inversion methods are also widely used in gravity inversion. Bosch et al. (2006) used Monte Carlo simulation techniques in gravity inversion. Chasseriau and Chouteau (2003) advocated 3-D inversion of gravity data using an *a priori* model of covariance. Shamsipour et al. (2010a,b, 2011) proposed a geostatic inversion technique for the inversion of gravity data obtained at ground surface and in boreholes.

The second kind of gravity inversion is probability tomography, which deals with the subsurface distribution of contrasting densities in a purely probabilistic sense without external constraints. Probability tomography was first developed for the analysis of self-potential data (Patella, 1997a,b) and then extended to geoelectric and electromagnetic methods (Mauriello and Patella, 1999a,b; Mauriello et al., 1998). Gravity and magnetic probability tomography imaging have also been developed as potential methods (Chianese and Lapenna, 2007; Guo et al., 2011a,b; Mauriello and Patella, 2001, 2005, 2008).

In gravity or magnetic inversion, non-uniqueness of the solution poses a problem for the mathematical properties of potential fields, in that many subsurface density or magnetic distributions produce identical responses. Inversion methods always need to impose constraints to help guide the inversion and obtain robust results (Bosh and McGaughey, 2001; Boulanger and Chouteau, 2001; Fullagar et al., 2008).

In comparison with methods that directly invert density contrast, probability tomography of gravity data is simple, stable and readily performed. Since the results are probabilities and not actual density contrasts, they take values between $-1$ and $+1$; it is also difficult to add geological constraints in the image processing to overcome the inherent non-uniqueness and to improve resolution. In this paper, we propose an iterative method of density contrast inversion that incorporates probability tomography computation of the mismatch between the observed

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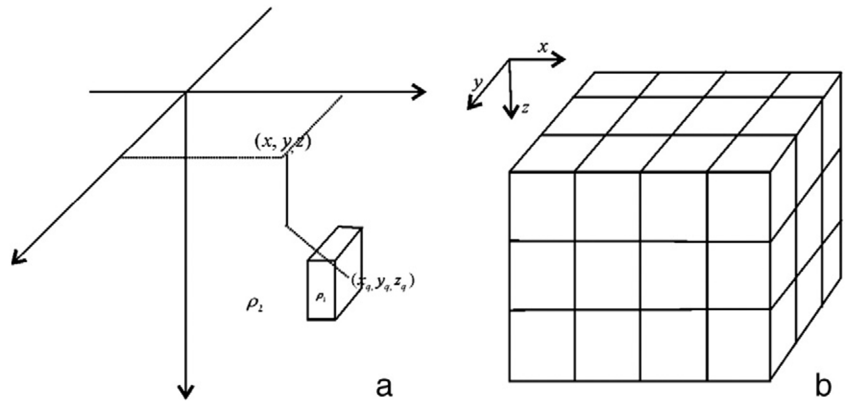


**Fig. 1.** (a) Rectangular cell $q$ of density $\rho_1$ within host volume of density $\rho_2$. (b) Division of subsurface into rectangular cells with constant density value.

gravity data and forward gravity data for a given density model. A straightforward constraint is also added to the inversion procedure that restricts the range of inverted density values to yield a geologically meaningful result.

## 2. Review of gravity tomography theory

In a reference Cartesian coordinate system ($x$–$y$) plane horizontal and $z$-axis positive downwards, the subsurface is divided into a large number of rectangular cells of constant density. Then, we assume that all gravity reading stations $P(x,y,z)$ are located on the ground surface at varying elevations $|z|$ above mean sea level. Referring to a cell $q$ with differential density $\Delta\rho_q$ with respect to the host material (Fig. 1a), the Bouguer anomaly $\Delta g_q(x,y,z)$ is written as:

$$\Delta g_q(x,y,z) = \frac{G\Delta\rho_q v_q\left(z_q - z\right)}{\left[\left(x_q - x\right)^2 + \left(y_q - y\right)^2 + \left(z_q - z\right)^2\right]^{\frac{3}{2}}} \tag{1}$$

where $G$ is the universal gravitational constant; $v_q$ is the volume of a rectangular cell; and $\Delta\rho_q = \rho_1 - \rho_2$ can be positive or negative, depending on the presence of an anomalous excess or deficit mass in cell $q$ with respect to the host volume. Summing all cells $Q$ in the region (Fig. 1b), the total Bouguer gravity anomaly $\Delta g(x_i,y_i,z_i)$ is given by:

$$\Delta g(x_i,y_i,z_i) = G\sum_{q=1}^{Q} \frac{\Delta\rho_q v_q\left(z_q - z_i\right)}{\left[\left(x_q - x_i\right)^2 + \left(y_q - y_i\right)^2 + \left(z_q - z_i\right)^2\right]^{\frac{3}{2}}} \tag{2}$$

where the subscripts $i$ of $x$, $y$, $z$ refer to a station point.

For a unit rectangular mass $\Delta\rho_q v_q = 1$, the Bouguer anomaly is expressed as:

$$\Delta g_u(x_i,y_i,z_i) = G\sum_{q=1}^{Q} \frac{\left(z_q - z_i\right)}{\left[\left(x_q - x_i\right)^2 + \left(y_q - y_i\right)^2 + \left(z_q - z_i\right)^2\right]^{\frac{3}{2}}} \tag{3}$$

Based on the derivation by Mauriello and Patella (2001), the probability tomography imaging function $\eta_q$ of a rectangular cell is obtained from:

$$\eta_q = \frac{\sum_{i=1}^{N} \Delta g(x_i,y_i,z_i)\Delta g_u(x_i,y_i,z_i)}{\sqrt{\sum_{i=1}^{N} \Delta g^2(x_i,y_i,z_i)\sum_{i=1}^{N} \Delta g_u^2(x_i,y_i,z_i)}} \tag{4}$$

where $N$ is the number of station points on the ground surface.

Substituting from Eqs. (3), (4) gives:

$$\eta_q = \frac{\sum_{i=1}^{N} \Delta g(x_i,y_i,z_i)B_q(x_i,y_i,z_i)}{\sqrt{\sum_{i=1}^{N} \Delta g^2(x_i,y_i,z_i)\sum_{i=1}^{N} B_q^2(x_i,y_i,z_i)}} \tag{5}$$

where:

$$B_q(x_i,y_i,z_i) = \frac{\left(z_q - z_i\right)}{\left[\left(x_q - x_i\right)^2 + \left(y_q - y_i\right)^2 + \left(z_q - z_i\right)^2\right]^{\frac{3}{2}}} \tag{6}$$

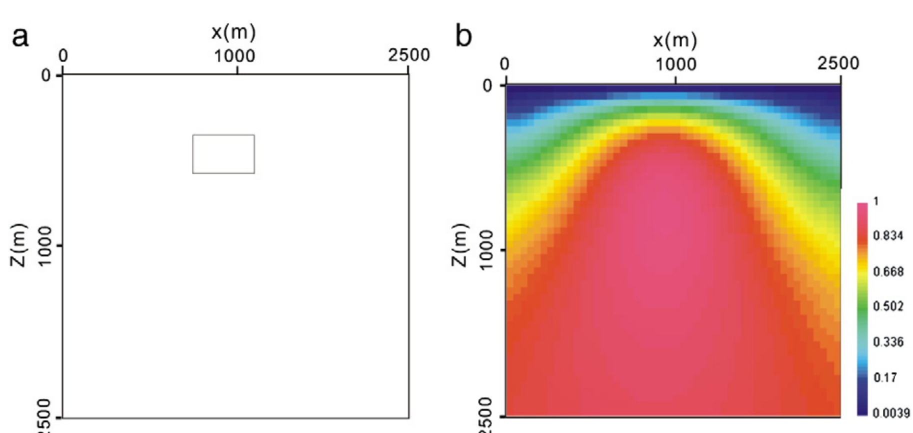


**Fig. 2.** (a) Geological model A consisting of a simple prism. (b) Probability imaging result for model A; the color scales represent probability value between −1 and +1.