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# Cylindrical-Wave Approach for electromagnetic scattering by subsurface metallic targets in a lossy medium

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ABSTRACT

An analytical solution is developed to the two-dimensional scattering problem of a plane-wave propagating in air, impinging on the interface with a dissipative soil, and interacting with a finite set of subsurface metallic targets. The Cylindrical Wave Approach is applied, the electromagnetic field scattered by the targets is expanded into cylindrical waves and use is made of the plane-wave spectrum to take into account the interaction of such waves with the planar interface between air and soil. The theoretical solution is implemented in a Fortran code. The numerical evaluation of the spectral integral relevant to reflected and transmitted cylindrical wave functions in the presence of lossy media is performed by means of Gaussian adaptive quadrature formulas. The method may return the field values in each point of the space, both in the near and far zones; moreover it may be applied for any polarization, and for arbitrary values of the cylinder sizes and positions.



## 1. Introduction

Detection of subsurface targets is a relevant topic in the frame of environmental sensing through radar techniques. Among the possible applications are the mapping of buried pipes and utilities, detection of voids and cavities within the ground, surveying of roads and pavements, and analysis of geological structures (Daniels, 2004; Jol, 2009). In these tasks, inverse scattering problems must be solved, in order to estimate the electromagnetic properties of the subsurface scenario from electromagnetic field measurements. Therefore, fast and accurate solvers of electromagnetic-scattering forward problems from known targets are fundamental in the analysis of the data collected by the radar (Frezza et al., 2011a; Meschino et al., 2010, 2012).

A two-dimensional (2D) geometry can be successfully employed for the modeling of scattering scenarios, if the sought buried structures are long with respect to the wavelength. Two half-spaces can be considered to represent the environment, where the upper one is filled with air and the lower one is soil hosting the buried targets. The soil can be modeled as a dielectric medium with complex permittivity and conductivity, and eventually also with a complex permeability. The targets, instead, can be modeled as dielectric or metallic cylinders.

Several techniques are proposed in the literature to solve the electromagnetic scattering problem from a single cylindrical scatterer buried in a dissipative medium. A buried cylinder of arbitrary cross-section is considered in Parry and Ward (1971), where a numerical solution of an integral equation for the electromagnetic field is proposed. In Mahmoud et al. (1981), solution to the scattering from a single circular cylinder in a dissipative half-space is given by means of a multipole expansion of the field generated by the current induced on the cylinder from an incident plane wave, but without an explicit solution for the field scattered in air. In Kanellopoulos and Buris (1984), the Green function method is applied to get an asymptotic solution for the scattered far field in air. In Sun et al. (2005), analytic solutions are developed for the single-scattering properties of an infinite dielectric cylinder embedded in an absorbing medium with normal incidence.

Many numerical results for the scattering by simple objects buried in dissipative media are reported in Brock and Sorensen (1994). A finite configuration of closely-spaced cylinders in an absorbing medium is considered in Lee (2011), and numerical results are given in terms of extinction and scattering cross-section.

In this paper, the 2D scattering problem from perfectly-conducting cylindrical targets in a lossy medium is solved by means of the Cylindrical Wave Approach (CWA). The CWA was previously developed as a spectral-domain solution to the plane-wave scattering from cylinders buried in a half-space with real permittivity (Di Vico et al., 2005a,b; Frezza et al., 2009b; Pajewski et al., 2009); scattering from a line-source was considered in Frezza et al. (2012a). The CWA was also extended to the time-domain in Frezza et al. (2007, 2008), where the incident field was a short-pulse plane wave. A combined use of CWA and Small Perturbation Method was presented in Fiaz et al. (2012), to solve the scattering from a cylinder below a slightly rough interface. The method was extended also to a geometry with the cylindrical

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objects buried in a dielectric slab between two half-spaces (Frezza et al., 2009a,b, 2010).

The fundamentals of the CWA are in the use of cylindrical waves as basis functions for the expansion of the fields scattered by the buried objects. A significant improvement in the method is presented in a recent work (Frezza et al., 2013), where the CWA was applied to solve scattering by one perfectly-conducting cylinder in a lossy half-space.

The proposed approach requires the use of Fresnel coefficients of complex argument to evaluate the interaction between the incident field, propagating in air, and the planar interface between air and lossy soil. The angles of both the phase vector and the attenuation vector of the transmitted wave, with respect to the normal to the interface, are evaluated by means of the method presented in Frezza and Tedeschi (2012) and Roy (2003). A scattered field is excited due to the interaction between the transmitted field and the buried objects; in its definition, all the cylinder/cylinder interactions are taken into account. On the basis of the plane-wave spectrum of a cylindrical function in a lossy medium developed in Frezza et al. (2011b), suitable reflected and transmitted cylindrical functions are defined which take into account the interaction between the field scattered by the cylinders and the planar air-soil interface. Results may be obtained for an arbitrary arrangement of the scatterers, both in transverse electric (TE) and transverse magnetic (TM) polarization states, as well as in near- or far-field zone.

In Section 2, the theoretical formulation of the problem is presented. In Section 3, the numerical results obtained through the implemented Fortran code are reported. Comparisons are made with commercial software based on Finite Element Method, emphasizing the advantages of our technique in terms of accuracy and computing time. Finally, in Section 4 conclusions are drawn.

## 2. Theory

A geometry with N perfectly conducting cylinders buried in a homogeneous, isotropic and lossy Medium 1, filling the half-space $x>0$, is considered (Fig. 1). An electromagnetic plane wave, propagating in the upper medium, i.e. the half-space $x<0$, impinges on the interface with the dielectric medium, with propagation vector $\mathbf{k}_i$ lying in the plane $(x, z)$. The upper half-space is supposed to be filled with air. The lower has relative electric permittivity $\varepsilon_1=\varepsilon_{1r}+i\varepsilon_{1i}$, with $\varepsilon_{1r}$, $\varepsilon_{1i}\in$R, and $\varepsilon_{1i}=\sigma/(\omega\varepsilon_0)$, being $\sigma$ as the medium conductivity, $\omega$ as the angular frequency, and $\varepsilon_0$ as the vacuum permittivity. Similarly, we define a complex refractive index $n_1=\sqrt{\varepsilon_1}$. The time factor $e^{-i\omega t}$ is omitted throughout the paper. Moreover, we suppose that both the media have the relative magnetic permeability equal to unity, because magnetic media are not generally involved in the considered applications. In any case, the presence of a magnetic medium would not significantly change the solution of the problem.

Within the Main Reference Frame (MRF) (O, $x$, $z$), a Reference Frame RF$_q$ centered on each q-th cylinder (q = 1,…,N), with both rectangular (O, $x_q$, $z_q$) and polar (O, $r_q$, $\theta_q$) coordinates, is considered. The q-th cylinder has a radius $a_q$ and center in ($d_q$, $h_q$) in Reference Frame RF$_q$. Normalized variables and parameters are used, defined as: $\xi_q=k_0x_q$, $\zeta_q=k_0z_q$, $\eta_q=k_0d_q$, $\chi_q=k_0h_q$, $\rho_q=k_0r_q$, and $\alpha_q=k_0a_q$, being $k_0=2\pi/\lambda_0$ as the vacuum wave number.

The electromagnetic field is expressed through the scalar quantity $V(x,z)$, which represents the $y$-component of the electric field in E-polarization (TM$^{(y)}$), and the $y$-component of the magnetic field in H-polarization (TE$^{(y)}$). The field $V$ is decomposed into different field contributions, which can be distinguished in plane-wave fields and scattered fields. Plane-wave fields are the incident field $V_i$ and the reflected and transmitted fields are $V_r$ and $V_t$, respectively. Scattered fields are the field $V_s$ scattered by the N cylinders inside Medium 1, and the fields $V_{sr}$ and $V_{st}$, i.e., the scattered reflected and scattered–transmitted fields, respectively.

The incident field $V_i$ has the following definition

$$V_i(\xi,\zeta)=V_0e^{i\left(n_\perp^i\xi+n_\parallel^i\zeta\right)} \tag{1}$$

where $n_\parallel^i$ and $n_\perp^i$ are the normalized parallel and orthogonal components of the wave vector.

The problem of the reflection and transmission at the interface is solved making use of Fresnel coefficients $\Gamma(n_\parallel)$ and $T(n_\parallel)$ of complex argument. The propagation vectors are divided in their real and imaginary parts, i.e. $\mathbf{k}=\boldsymbol{\beta}+i\boldsymbol{\alpha}$, and in the general case of propagation through the interface between two lossy media, can be evaluated using a generalization of the Snell's law (Roy, 2003). In particular, as to the reflected propagation vector $\mathbf{k}_r$, the behavior is the same as in reflection at an interface between two lossless media, i.e. with the same tangential component of the incident vector, and with a change of sign in the normal component. Therefore, the reflected field $V_r$ is the following

$$V_r(\xi,\zeta)=V_0\Gamma_{01}\left(n_\parallel^i\right)e^{i\left(-n_\perp^i\xi+n_\parallel^i\zeta\right)} \tag{2}$$

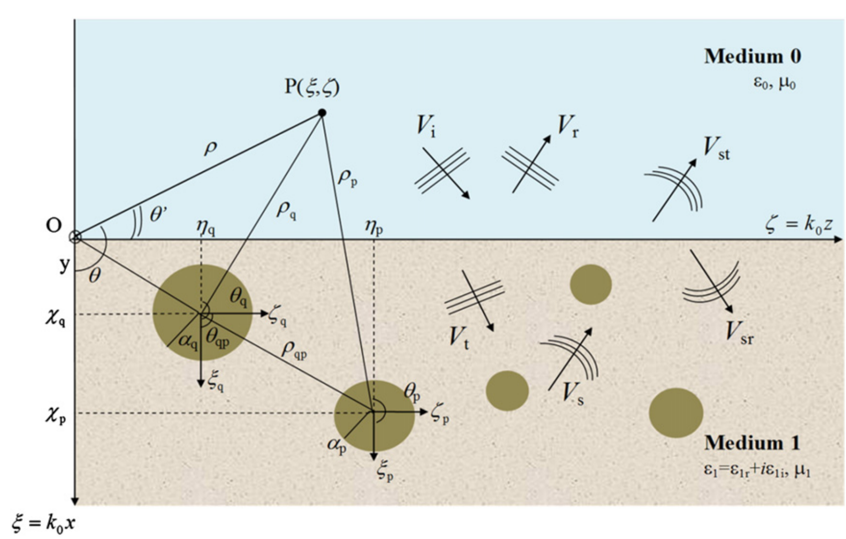


**Fig. 1.** Geometry of the problem.