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Application of balanced edge detection filters to estimate the location parameters of the causative sources using potential field data

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ABSTRACT

Balanced edge detection filters can recognize the edges of the shallow and deep bodies simultaneously, and are commonly used in the edge detection of potential field data. In this paper, we present using the balanced edge detection filters to estimate source locations, and derive two linear equations based on the balanced edge detection filters that can estimate the locations of the source without any priori information about the nature (structural index) of the source. The proposed methods are demonstrated on synthetic gravity anomalies, and the inversion results show that the proposed methods can successfully estimate location parameters of the sources. I also apply the proposed methods to real magnetic data, and the inversion results estimated by the proposed methods are consistent with the results estimated by the other similar method.

1. Introduction

Edge detection of potential field data has been widely used as an indispensable tool in exploration technology. At first, some authors used various combinations of the horizontal and vertical derivatives to recognize the edges of the sources (Blakely, 1995; Cordell and Grauch, 1985; Evjen, 1936; Nabighian, 1984; Roest et al., 1992; Thurston and Smith, 1997), and these methods can recognize the edges of shallow bodies, but they cannot obtain the edges of the deep bodies clearly. To solve this disadvantage, some people presented the balanced edge detection filters that can display the edges of shallow and deep bodies simultaneously (Cooper, 2009; Cooper and Cowan, 2006, 2008; Ma, 2013; Ma and Li, 2012; Miller and Singh, 1994; Verduzco et al., 2004; Wijns et al., 2005), and the *TDX* and *Theta* map are two commonly used balanced edge detection filters, but the edge detection filters only can provide the information about the horizontal location of the source.

One of important targets in the interpretation of potential field data is to compute the depths of the sources. To this end, we derive two equations based on the *TDX* and *Theta* map filters to estimate the horizontal location and depth of the source without any priori information about the nature (structural index) of the source.

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2. Methodology

Cooper and Cowan (2006) presented the normalized tilt angel (*TDX*) filter, which can be given by

$$TDX = \tan^{-1}\left(\frac{\sqrt{(\partial f/\partial x)^2 + (\partial f/\partial y)^2}}{\partial f/\partial z}\right) \quad (1)$$

where, f is the original gravity or magnetic anomaly. Computing the derivatives of Eq. (1) in the x, y and z directions, we get

$$\frac{\partial TDX}{\partial x} = \frac{1}{A^2}\left(\frac{\partial f}{\partial z}\frac{1}{THD}\times\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y}\right) - THD \times \frac{\partial^2 f}{\partial x \partial z}\right) \quad (2)$$

$$\frac{\partial TDX}{\partial y} = \frac{1}{A^2}\left(\frac{\partial f}{\partial z}\frac{1}{THD}\times\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right) - THD \times \frac{\partial^2 f}{\partial y \partial z}\right) \quad (3)$$

$$\frac{\partial TDX}{\partial z} = \frac{1}{A^2}\left(\frac{\partial f}{\partial z}\frac{1}{THD}\times\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial z} + \frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y \partial z}\right) - THD \times \frac{\partial^2 f}{\partial z^2}\right) \quad (4)$$

where, $THD = \sqrt{(\partial f/\partial x)^2 + (\partial f/\partial y)^2}$ is the total horizontal derivative of the data, and $A = \sqrt{(\partial f/\partial x)^2 + (\partial f/\partial y)^2 + (\partial f/\partial z)^2}$ is the analytic signal of the data.

The 3D form of the Euler equation for a potential field f can be given by (Reid et al., 1990; Thompson, 1982)

$$(x-x_0)\frac{\partial f}{\partial x}+(y-y_0)\frac{\partial f}{\partial y}+(z-z_0)\frac{\partial f}{\partial z}=-N(f-B). \tag{5}$$

where, x, y and z are the observation coordinates, x_0, y_0, and z_0 are the source coordinates, B is the background field, and N is the structural index characterizing the nature of the source. The solution of Euler equation requires the information about the structural index of the source, but the structural index is not unique and is hard to know for an unknown area, which is a constraint of conventional Euler deconvolution method.

Computing the derivatives of Eq. (5) in the x, y and z directions and assuming a constant background B, and we can obtain

$$(x-x_0)\frac{\partial^2 f}{\partial x^2}+(y-y_0)\frac{\partial^2 f}{\partial x\partial y}+(z-z_0)\frac{\partial^2 f}{\partial x\partial z}=-(N+1)\frac{\partial f}{\partial x} \tag{6}$$

$$(x-x_0)\frac{\partial^2 f}{\partial x\partial y}+(y-y_0)\frac{\partial^2 f}{\partial y^2}+(z-z_0)\frac{\partial^2 f}{\partial y\partial z}=-(N+1)\frac{\partial f}{\partial y} \tag{7}$$

$$(x-x_0)\frac{\partial^2 f}{\partial x\partial z}+(y-y_0)\frac{\partial^2 f}{\partial y\partial z}+(z-z_0)\frac{\partial^2 f}{\partial z^2}=-(N+1)\frac{\partial f}{\partial z}. \tag{8}$$

Multiplying Eqs. (6), (7) and (8) by $\frac{\frac{\partial f \partial f}{\partial x \partial z}}{THD}$, $\frac{\frac{\partial f \partial f}{\partial y \partial z}}{THD}$ and THD, respectively, and adding the first two and then subtracting the third one, we get

$$\begin{aligned}(x-x_0)&\left[\frac{1}{THD}\frac{\partial f}{\partial z}\times\left(\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial y}\right)-THD\frac{\partial^2 f}{\partial x\partial z}\right]\\ +(y-y_0)&\left[\frac{1}{THD}\frac{\partial f}{\partial z}\times\left(\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}\right)-THD\frac{\partial^2 f}{\partial y\partial z}\right]\\ +(z-z_0)&\left[\frac{1}{THD}\frac{\partial f}{\partial z}\times\left(\frac{\partial^2 f}{\partial x\partial z}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y\partial z}\frac{\partial f}{\partial y}\right)-THD\frac{\partial^2 f}{\partial z^2}\right]=0.\end{aligned} \tag{9}$$

Substitute Eqs. (2), (3) and (4) into Eq. (9), we obtain

$$(x-x_0)\frac{\partial TDX}{\partial x}+(y-y_0)\frac{\partial TDX}{\partial y}+(z-z_0)\frac{\partial TDX}{\partial z}=0. \tag{10}$$

We derive a linear equation based on the *TDX* filter to estimate the location parameters of the causative source.

Wijns et al. (2005) introduced the *Theta* map, which is based on the analytic signal, and can be written by

$$Theta=\frac{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}}{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2+\left(\frac{\partial f}{\partial z}\right)^2}}. \tag{11}$$

Computing the derivatives of Eq. (11) in the x, y and z directions, we obtain

$$\frac{\partial Theta}{\partial x}=\frac{1}{A^2}\left(\frac{1}{THD}\left(\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial y}\right)\times A-THD\times\frac{\partial A}{\partial x}\right) \tag{12}$$

$$\frac{\partial Theta}{\partial y}=\frac{1}{A^2}\left(\frac{1}{THD}\left(\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}\right)\times A-THD\times\frac{\partial A}{\partial y}\right) \tag{13}$$

$$\frac{\partial Theta}{\partial z}=\frac{1}{A^2}\left(\frac{1}{THD}\left(\frac{\partial^2 f}{\partial x\partial z}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y\partial z}\frac{\partial f}{\partial y}\right)\times A-THD\times\frac{\partial A}{\partial z}\right). \tag{14}$$

Multiplying Eqs. (6) and (7) by $\frac{\partial f/\partial x}{THD\tilde{n}A}$ and $\frac{\partial f/\partial y}{THD\tilde{n}A}$, respectively, and then adding the first and second one, we get

$$\begin{aligned}&(x-x_0)\frac{\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial y}}{THD\times A}+(y-y_0)\frac{\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}}{THD\times A}\\ &+(z-z_0)\frac{\frac{\partial^2 f}{\partial x\partial z}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y\partial z}\frac{\partial f}{\partial y}}{THD\times A}=-(N+1)\frac{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}{THD\times A}.\end{aligned} \tag{15}$$

Huang et al. (1995) showed that if an original anomaly is homogeneous of degree N, the analytic signal of the anomaly is homogeneous of degree $N+1$, so the Euler equation of analytic signal becomes

$$(x-x_0)\frac{\partial A}{\partial x}+(y-y_0)\frac{\partial A}{\partial y}+(z-z_0)\frac{\partial A}{\partial z}=-(N+1)A. \tag{16}$$

Multiplying Eq. (16) by $\frac{THD}{A^2}$, and subtracted by Eq. (15), we obtain

$$\begin{aligned}&(x-x_0)\left(\frac{\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial y}}{THD\times A}-\frac{\partial A/\partial x}{A^2}\times THD\right)\\ &+(y-y_0)\left(\frac{\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}}{THD\times A}-\frac{\partial A/\partial y}{A^2}\times THD\right)\\ &+(z-z_0)\left(\frac{\frac{\partial^2 f}{\partial x\partial z}\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial y\partial z}\frac{\partial f}{\partial y}}{THD\times A}-\frac{\partial A/\partial z}{A^2}\times THD\right)=0.\end{aligned} \tag{17}$$

Substitute Eqs. (12), (13) and (14) into Eq. (17), we obtain

$$(x-x_0)\frac{\partial Theta}{\partial x}+(y-y_0)\frac{\partial Theta}{\partial y}+(z-z_0)\frac{\partial Theta}{\partial z}=0. \tag{18}$$

We can estimate the location parameters x_0, y_0, and z_0 of the source by solving Eqs. (10) and (18) based on the *TDX* and *Theta* map filters.

In applying the proposed methods, we use a four steps clustering method to get an accurate horizontal location and depth of the causative source. The first clustering is that the distance between the estimated horizontal locations and the horizontal location of observed point is less than half of the window size. The reason is that we can obtain more accurate results when the observed points above the source (Reid et al., 1990). The second clustering method is done on a small window to remove isolated solutions. It is based on the regulation that a point belongs to a cluster if the distance between that point and all the other points belonging to the cluster is smaller than a threshold value not bigger than the size of the window. The third clustering step identifies the already focused solutions by the previous clustering method in more general clusters and achieves a fusion of the clusters. It identifies as belonging to the same cluster all points whose horizontal center is less than the maximum horizontal radius of confidence of all the clusters. Finally, we apply a filter that removes the clusters with less than a given number of solutions, because the clusters with a small number of solutions are statistically not significant.

3. Tests on synthetic potential field data

To test the performance of the proposed methods, we apply them to a synthetic gravity anomaly generated by a rectangular prism. The prism is located at (50, 50) m with a square size of 24 m and a top depth of 5 m, and the gravity anomaly is computed with an interval of 1 m. Fig. 1a shows the gravity anomaly of the prism. Fig. 1b and c