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Improvements in crosshole GPR full-waveform inversion and application on data measured at the Boise Hydrogeophysics Research Site

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ABSTRACT

Crosshole ground penetrating radar (GPR) tomography has been widely used and has the potential to improve the obtained subsurface models due to its high spatial resolution compared to other methods. Recent advances in full-waveform inversion of crosshole GPR data show that higher resolution images can be obtained compared to conventional ray-based GPR inversion because it can exploit all information present in the observed data. Since the first application of full-waveform inversion on synthetic and experimental GPR data, the algorithm has been significantly improved by extending the scalar to a vectorial approach, and changing the stepped permittivity and conductivity update into a simultaneous update. Here, we introduce new normalized gradients that do not depend on the number of sources and receivers which enable a comparison of the gradients and step lengths for different crosshole survey layouts. An experimental data set acquired at the Boise Hydrogeophysics Research Site is inverted using different source–receiver setups and the obtained permittivity and conductivity images, remaining gradients and final misfits are compared for the different versions of the full-waveform inversion. Moreover, different versions of the full-waveform inversion are applied to obtain an overview of all improvements. Most improvements result in a reducing final misfit between the measured and synthetic data and a reducing remaining gradient at the final iteration. Regions with relatively high remaining gradient amplitudes indicate less reliable inversion results. Comparison of the final full-waveform inversion results with Neutron–Neutron porosity log data and capacitive resistivity log data show considerably higher spatial frequencies for the logging data compared to the full-waveform inversion results. To enable a better comparison, we estimated a simple wavenumber filter and the full-waveform inversion results show an improved fit with the logging data. This work shows the potential of full-waveform inversion as an advanced method that can provide high resolution images to improve hydrological models.

1. Introduction

Crosshole ground penetrating radar (GPR) tomography can provide a high resolution profile of the shallow subsurface electrical properties (dielectric permittivity ε and electrical conductivity σ) between two boreholes (Holliger et al., 2001; Irving and Knight, 2005; Tronicke et al., 2002). For crosshole GPR surveys, tomographic inversions are generally based on geometrical ray theory (Dafflon and Barrash, 2012; Dafflon et al., 2011; Irving et al., 2007; Maurer and Musil, 2004). It provides electromagnetic velocity and attenuation images of the probed regions by first-arrival times and maximum first-cycle amplitude inversions. Conventional ray tomography can suffer from critical shortcomings associated with the limitation of the high-frequency approximation, the limited angular coverage of the target, and the limited information present in the observed signal that is employed in the inversion process. Furthermore, ray-based inversion usually only resolves features larger than the dominant signal wavelength (resolution scales approximately with the diameter of the first Fresnel zone) and it cannot provide reliable information on certain important types of low-velocity (high-permittivity) structures (Williamson and Worthington, 1993).

The resolution of the images can be significantly improved by using a full-waveform inversion (FWI) that considers the entire waveform or significant parts thereof (Ernst et al., 2007a). The FWI has been first proposed in exploration seismology and has been developed for both acoustic and elastic waves generated and recorded at the surface or borehole. The FWI provides sub-wavelength resolution and reliable information on a broad range of structures, including those distinguished by low velocities. To determine an update of the medium properties, the full-waveform modeling is performed at each iteration by using finite-difference or finite-element approaches that can be performed in either the time- or frequency domain (Pratt, 1990, 1999; Tarantola, 1984, 1986; Virieux and Operto, 2009; Zhou and Greenhalgh, 2003). One of the first FWIs of crosshole GPR approach was applied to synthetic

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(Ernst et al., 2007a), and two experimental data sets (Ernst et al., 2007b); one obtained in the Grimsel rock laboratory and one obtained at the Boise Hydrogeophysics Research Site (BHRS) near Boise, Idaho, USA. Since these first applications of FWI for crosshole GPR data, the FWI has been improved by including the vector character of the electrical field and the simultaneous inversion of permittivity and conductivity (Meles et al., 2010). The improved FWI approach also was optimized and applied to an experimental data set near the River Thur in Switzerland (Klotzsche et al., 2010). The full-waveform inversion was able to reconstruct a low-velocity waveguide layer, which was caused by an increase in porosity and indicates a zone of preferential flow within the aquifer (Klotzsche et al., 2012). Compared to traditional hydrological measurements such as borehole logging and petrophysical analysis, crosshole GPR tomography provides field-scale information of the shallow subsurface. This information can improve soil water content estimation and flow change. In this respect, crosshole GPR tomography has been widely used in hydrology and showed its potential for aquifer characterization (Binley et al., 2001, 2002; Deiana et al., 2007; Hubbard et al., 1997; Looms et al., 2008; Slater et al., 1997; Winship et al., 2006). The permittivity values can be converted to effective porosity by empirical formulas such as Topp et al. (1980) and Linde et al. (2006) and hydraulic conductivity by using geostatistics and Kozeny–Carman relation (Gloaguen et al., 2001).

In the following, an overview is given of all developments by showing the improvements of the FWI to the experimental data set acquired at the BHRS. Moreover, four times as many receivers are used to investigate the improvements. Because the misfit and gradients initially were depending on the number of sources and receivers, we first introduce a normalized misfit and gradient that is independent on the number of source and receivers. After introducing the BHRS and the crosshole GPR setup, the inversion results and remaining gradients are compared between the ray-based and different version of the FWI. Finally, we evaluate the FWI results with Neutron–Neutron porosity and capacitive resistivity logging data. Due to the different spatial resolutions, a wavenumber filter is introduced that enables a better comparison.

2. Full-waveform inversion methodology

In the earlier years of the 1980, it was a challenge to invert experimental data sets measured with a large number of sources and receivers, due to the limited computing resources available. Due to the recent developments of parallel programming tools on massive parallel computer structures, considerable effort has been dedicated to develop techniques that allow solving problems involving large numbers of parameters. The FWI minimizes the full-waveform differences between the synthetic GPR data and the observed GPR data at the receiver positions for all source–receiver pairs of the GPR survey by updating the spatial distributions of the medium properties ε and σ. The misfit between the recorded and modeled data is described by the squared misfit norm $S(\varepsilon,\sigma)$:

$$S(\varepsilon,\sigma) = \frac{1}{2}\sum_{s}\sum_{r}\sum_{\tau}\left[E^{s}_{\text{syn}}(\varepsilon,\sigma) - E^{s}_{\text{obs}}\right]^{T}_{r,\tau} \cdot \left[E^{s}_{\text{syn}}(\varepsilon,\sigma) - E^{s}_{\text{obs}}\right]_{r,\tau}, \qquad (1)$$

where $E^{s}_{\text{syn}}(\varepsilon,\sigma)$ and E^{s}_{obs} are the synthetic and observed data, respectively, and T denotes the adjoint operator (transpose conjugate). Note that we use here Meles et al. (2010) formalism where $E^{s}_{\text{syn}}(\varepsilon,\sigma)$ and E^{s}_{obs} contain the data for all sources, receivers and observation times, such that the sum over sources s, receivers r and observation time τ in Eq. (1) returns the overall misfit to be minimized. Since the full-waveform is present within the observation time we need an accurate forward model that solves the full-waveform results of Maxwell's equations for all source–receiver combinations. Here, the FWI of crosshole GPR data is based on a 2D finite-difference time-domain (FDTD) solutions of Maxwell's equations. The medium properties ε and σ are updated using the following recipe:

1) Select initial models $\varepsilon = \varepsilon_{\text{ini}}$ and $\sigma = \sigma_{\text{ini}}$ (usually obtained by ray-based tomography results).
2) Compute synthetic wave fields at the receiver positions using the initial models.
3) Compute the residual wave field by subtracting the synthetic from the measured data.
4) Compute the gradient directions ∇S_{ε} and ∇S_{σ} by a cross-correlation of the synthetic wave field with the back-propagated residual wave field. Here, the cross-correlation can be scalar by only including the vertical electric wave fields, or vectorial by including the vertical and horizontal electric wave fields.
5) Compute the update directions d_{ε} and d_{σ} with the conjugate gradient (CG) method using the gradient directions ∇S_{ε} and ∇S_{σ}.
6) Compute the step lengths ζ_{ε} and ζ_{σ} using a linear step length calculation and carefully chosen perturbation factors that cannot be too large to make sure the perturbed model still lies in the linearity range and inversion overshooting is avoided, and not too small to avoid truncation (round-off) errors when dealing with small numbers (Meles et al., 2010).
7) Update the model parameters ε and σ using

$$\begin{aligned}\varepsilon^{(k+1)} &= \varepsilon^{(k)} - \zeta^{(k)}_{\varepsilon} d^{(k)}_{\varepsilon}\\ \sigma^{(k+1)} &= \sigma^{(k)} - \zeta^{(k)}_{\sigma} d^{(k)}_{\sigma},\end{aligned} \qquad (2)$$

where k is the iteration number. Here, the approach can be the stepped or cascaded by alternately updating one parameter for certain number of iterations while keeping the other one fixed, or simultaneous by updating both parameters in one iteration.

8) Repeat steps 2 through 7 until convergence has been achieved. Usually, when the remaining residuals are less than 1%, this indicates that the inversion is converged and returns credible results.

The gradients of the misfit function with respect to permittivity and conductivity ∇S_{ε} and ∇S_{σ} are obtained by cross-correlating the incident wave field emitted from the source with the residual wave fields that are back-propagating from the receiver at all medium locations in time domain for all source and receiver combinations as follows (see Eq. (23) in Meles et al., 2010):

$$\begin{bmatrix}\nabla S_{\varepsilon}\left(x'\right)\\ \nabla S_{\sigma}\left(x'\right)\end{bmatrix} = \sum_{s}\sum_{r}\sum_{\tau}\begin{matrix}\left(\partial_t E^{s}\right)^{T}\widehat{G}^{T}\left[\Delta E^{s}\right]_{r,\tau}\\ \left(E^{s}\right)^{T}\widehat{G}^{T}\left[\Delta E^{s}\right]_{r,\tau}\end{matrix}, \qquad (3)$$

where ΔE^{s} is the residual wave field at the receiver positions and $\widehat{G}^{T}$ is the back-propagation operator. Note that the ε and σ gradients only differ for a time derivative.

Because of vertical dipole-type transmitter antennas being used in crosshole GPR, the first version of FWI (Ernst et al., 2007b; Belina et al., 2012a, b) used only the E_z component of the electric field for the calculation of the gradient and ignored the E_x component. Therefore, we call it in the following scalar FWI. However, for large vertical distances between the source and receiver positions, the E_x components can significantly contribute in the gradient calculation and should therefore be included to honor the vectorial character of the electromagnetic waves. Accordingly, we refer to this version as vector FWI (Meles et al., 2010), which can also be used to invert borehole to surface data or four-sided inversions (Meles et al., 2011).

The first FWI version initially used a stepped or cascaded approach where the permittivities were updated for a certain number of iterations while keeping the conductivities fixed and then analogously the conductivities were inverting while keeping the permittivities fixed (Ernst et al., 2007a). However, Eq. (4) indicates that at each iteration both the permittivity and conductivity gradients can be calculated. To obey the simultaneous nature of the electromagnetic wave propagation, a new simultaneous version FWI has been proposed that simultaneously updates the permittivities and conductivities at each iteration (Meles