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Tilt angle interpretation of dipping fault model

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ABSTRACT

The interpretation of tilt angle transformed total magnetic intensity data for a dipping contact model is studied, where it is shown that if the trace of a dipping contact on a horizontal surface is determined then other source parameters, such as the dip and the depth of burial of the contact can be estimated using simple formulas. The trace of the dipping contact can be obtained from the location of the peak of a profile of the first order horizontal derivative of a tilt angle. The tilt angle response and its horizontal derivative are found to be sensitive to noise; however, it is shown that upward continuation operation which does not alter the location of the trace of the contact can improve the tilt angle interpretation for the measured noisy data. Use on synthetic model studies and subsequently on field examples on an aeromagnetic profile across the San Ysidro Fault in the Rio Grande Ridge, U.S.A., and also on an extracted profile from 2D ground magnetic data across Son-Narmada Fault in India demonstrates the applicability of the tilt angle interpretation to delineate the subsurface architecture.

1. Introduction

Tilt angle (Miller and Singh, 1994), a relatively newly defined attribute of potential field data is becoming increasingly popular among geoscientists for interpreting potential field, especially, the aeromagnetic data. The power of the tilt angle in delineating body edges and, especially, the linear structures from a high resolution magnetic anomaly image in the tilt angle domain has been demonstrated by Thurston and Smith (1997), Verduzco et al. (2004), Salem et al. (2007, 2008), and Lahti and Karinen (2010). Verduzco et al. (2004) explained that high resolution attained in a tilt angle image is mainly due to a high dynamic gain attributed by the natural property of the arctangent function, the value of which ranges between $-90°$ and $90°$. The tilt angle and its horizontal derivative are termed alternatively in the literature as local phase and local wavenumber respectively (Pilkington and Keating, 2006; Thurston and Smith, 1997; Verduzco et al., 2004) as those are closely associated with analytical signal definition of a potential field anomaly. In this paper, I use the terms tilt angle (TA) and horizontal derivative of tilt angle (HDTA).

Buried fault is a common geologic structure of interest in many geophysical applications, whose signature is often subtle in total magnetic intensity images, where TA, which is a derivative field, should be an important tool for interpretation. Both normal and reverse magnetic anomaly of a fault can be modeled with a contact model. Salem et al. (2007) showed that TA offers a simple method of identifying and determining the depth of buried vertical contact, where the location of 0° of the quantity TA corresponds to the trace of the vertical contact, and the distance between the locations of $-45°$ and $45°$ on the TA profile corresponds to twice the depth of burial of the contact. Recently, Salem et al. (2010) used TA to prepare the depth to the basement map using the assumption of a vertical contact model.

In this paper, I show with numerical experiments major implications of interpreting a TA profile over a dipping buried contact model. First, the location of zero value of the TA profile does not correspond to the trace of the obliquely dipping contact. Second, the trace of the contact can be determined from the location of the peak of HDTA profile, which is independent of the dip of the contact. Third, TA anomaly is strongly sensitive to the noise in TMI measurement. And lastly, upward continuation improves interpretability of noise contaminated TA anomaly. The above aspects are demonstrated via numerical experiments on synthetic data. The applications of tilt angle interpretation on aeromagnetic profile of TMI data across the San Ysidro Fault of Rio Grande Ridge, U.S.A. and on a vertical magnetic profile extracted from a magnetic map of Son-Narmada Rift Valley of central India are presented.

2. Theoretical background

2.1. Formulation

Consider a contact model in Fig. 1 with thickness t striking along the Y-axis in a 3-axis Cartesian coordinate frame with origin O, where X-Z is a 2D vertical section along the measurement profile on a horizontal surface directed along the X-axis. Suppose that the contact is buried at a depth h from the horizontal surface with a dip θ from the horizontal.

The depth of burial of the contact is assumed to be many times smaller than its horizontal extent, so that the contact model can be considered a 2D source. I assume that the magnetic polarization is mainly due to induced magnetization; if remnant magnetization at all exists

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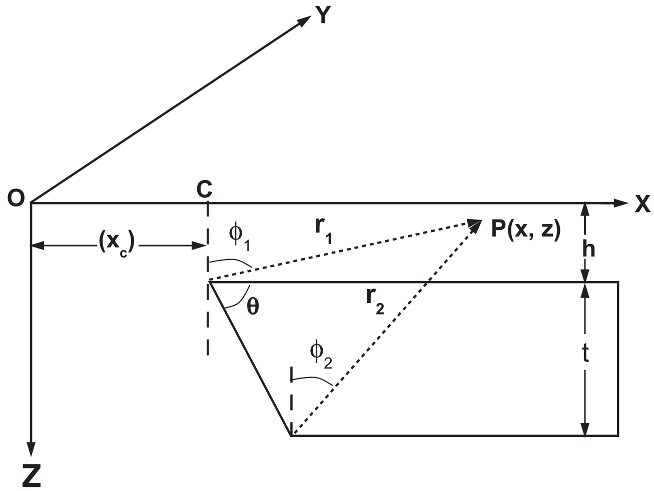

Fig. 1. Schematic diagram of the dipping contact model.

then it is either parallel to the direction of induced magnetization or its magnitude is negligibly small compared to that of induced polarization. The magnitude, inclination and declination of the induced magnetic field are F, i and D respectively. Consider $P(x, z)$ on the X-Z plane is a point of measurement, where Z-axis is oriented positive downward. The profiles of horizontal and vertical derivatives of the measured total magnetic intensity (TMI) $T(x)$ when the thickness $t \to \infty$ are given as (Nabighian, 1972):

$$T_x(x) = \frac{\partial T}{\partial x} = 2\kappa F\eta \sin\theta \frac{(h-z)\cos(2I-\theta-90^\circ) + (x-x_c)\sin(2I-\theta-90^\circ)}{(h-z)^2 + (x-x_c)^2}, \quad (1)$$

$$T_z(x) = \frac{\partial T}{\partial z} = 2\kappa F\eta \sin\theta \frac{(x-x_c)\cos(2I-\theta-90^\circ) - (h-z)\sin(2I-\theta-90^\circ)}{(h-z)^2 + (x-x_c)^2}, \quad (2)$$

where,

$$\tan I = \frac{\tan i}{\cos\xi}, \xi \text{ being the angle between magnetic north and the positive X-axis,} \quad (3)$$

$$\eta = 1 - \cos^2 i \sin^2\xi \quad (4)$$

κ is the magnetic susceptibility contrast.

The pole reduction operation on TMI data, even when TMI data are available from a low latitude area is necessary before any further analysis, as the reduced-to-pole (RTP) transformation removes asymmetry on the shape of the TMI anomaly caused by the direction of magnetization of ambient magnetic field (see for example, Silva (1986) and Li and Oldenburg (2001)). It then turns out from Eqs. (3) and (4), that I is equal to 90° and η is equal to 1, in which case Eqs. (1) and (2) become

$$T_x(x) = 2\kappa F \sin\theta \frac{(h-z)\sin\theta + (x-x_c)\cos\theta}{(h-z)^2 + (x-x_c)^2}, \quad (5)$$

$$T_z(x) = 2\kappa F \sin\theta \frac{(x-x_c)\sin\theta - (h-z)\cos\theta}{(h-z)^2 + (x-x_c)^2}. \quad (6)$$

Using Eqs. (5) and (6) the TA as defined by Miller and Singh (1994) can be written as

$$\delta(x) = \arctan\left[\frac{(x-x_c)\sin\theta - (h-z)\cos\theta}{|(h-z)\sin\theta + (x-x_c)\cos\theta|}\right], \quad (7)$$

where, $|\cdot|$ denotes the absolute value. Ensuring positive value of the denominator in Eq. (7) is essential, as $\delta(x) \in [-90^\circ, 90^\circ]$. Since, we normally measure magnetic anomaly on the horizontal surface ($z = 0$) Eq. (7) can be rewritten as

$$\delta(x) = \arctan\left[\frac{(x-x_c)\sin\theta - h\cos\theta}{|(x-x_c)\cos\theta + h\sin\theta|}\right]. \quad (8)$$

Note that with Eq. (8), the TA response $\delta(x)$ for a buried dipping contact model is independent of magnetic field or magnetization. With the Eq. (8) following properties of $\delta-$ profile across a buried contact model becomes immediately apparent:

For a nearly outcropping contact ($h \to 0$)

$$\tan\delta(x) = \tan\theta. \quad (9)$$

It then turns out from Eq. (9) that TA profile across nearly outcropping dipping contact would remain flat with its value equal to the dip of the contact, where the numeric values of θ lie between 0° and 90°, having an appropriate sign convention. For example, if θ varies between 90° and 180° the numeric value of θ will be $\theta - 90°$ but with a negative sign. On the other hand, $\theta \le 90°$ will correspond to a positive value of θ. However, when $h > 0$, the $\delta-$ profile becomes a stretched S-type curve. It attains the value 0° when the numerator within the square bracket of Eq. (8) becomes zero, which implies

$$\tan\theta = \frac{h}{x_0 - x_c}, \quad \text{or}, \quad x_0 - x_c = h\cot\theta. \quad (10)$$

At a location of the trace of the contact on a horizontal surface, i.e., at $x = x_c$

$$\tan\delta(x_c) = \tan(\theta - 90^\circ) \Rightarrow \delta(x_c) = \theta - 90^\circ. \quad (11)$$

It follows from Eq. (11) $\delta(x) = 0°$, for $\theta = 90°$ at a location of the trace of the contact. For a buried vertical contact, the trace of the contact on the horizontal surface corresponds to the zero value of the TA. However, the location of the zero value of a TA profile moves away from the trace of the dipping contact on a horizontal surface by an amount $h\cot\theta$ either in the positive X-direction when $\theta \in (0°,90°)$ or in the negative X-direction when $\theta \in (180°,90°)$.

Since, the value of $\delta(x)$ lies between $-90°$ and 90° then from Eq. (8) $\delta(x)$ will attain $\pm 90°$ only when the expression inside the arctangent becomes infinity which is achieved when the denominator of the expression tends to zero. Hence,

$$|(x-x_c)\cos\theta + h\sin\theta| = 0, \quad \text{which implies} \quad x = x_c - h\tan\theta. \quad (12)$$

Over a finite length of a TA profile, if $0 < \theta < 90°$ then $x_{-90^\circ} = x_c - h \tan\theta$, where x_{-90° corresponds to the location of $-90°$ value of the TA, and x_{-90° trails x_c. On the other hand, if $90° < \theta < 180°$ then $x_{90^\circ} = x_c - h \tan\theta$, where x_{90} corresponds to the location of 90° value of TA, and x_{90} leads x_c, as $\tan\theta$ will be negative. However, for a vertical contact $\delta(x)$ attains $\pm 90°$ only at the ends of a substantially long profile. Using Eqs. (10) and (12) it can be shown that

$$h = \left[(x_c - x_{\pm 90^\circ})(x_0 - x_c)\right]^{1/2}. \quad (13)$$