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An implementation of differential evolution algorithm for inversion of geoelectrical data

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ABSTRACT

Differential evolution (DE), a population-based evolutionary algorithm (EA) has been implemented to invert self-potential (SP) and vertical electrical sounding (VES) data sets. The algorithm uses three operators including mutation, crossover and selection similar to genetic algorithm (GA). Mutation is the most important operator for the success of DE. Three commonly used mutation strategies including DE/best/1 (strategy 1), DE/rand/1 (strategy 2) and DE/rand-to-best/1 (strategy 3) were applied together with a binomial type crossover. Evolution cycle of DE was realized without boundary constraints. For the test studies performed with SP data, in addition to both noise-free and noisy synthetic data sets two field data sets observed over the sulfide ore body in the Malachite mine (Colorado) and over the ore bodies in the Neem-Ka Thana cooper belt (India) were considered. VES test studies were carried out using synthetically produced resistivity data representing a three-layered earth model and a field data set example from Gökçeada (Turkey), which displays a seawater infiltration problem. Mutation strategies mentioned above were also extensively tested on both synthetic and field data sets in consideration. Of these, strategy 1 was found to be the most effective strategy for the parameter estimation by providing less computational cost together with a good accuracy. The solutions obtained by DE for the synthetic cases of SP were quite consistent with particle swarm optimization (PSO) which is a more widely used population-based optimization algorithm than DE in geophysics. Estimated parameters of SP and VES data were also compared with those obtained from Metropolis–Hastings (M–H) sampling algorithm based on simulated annealing (SA) without cooling to clarify uncertainties in the solutions. Comparison to the M–H algorithm shows that DE performs a fast approximate posterior sampling for the case of low-dimensional inverse geophysical problems.

1. Introduction

Geoelectrical methods including self-potential (SP) and direct current (DC) electrical resistivity based on vertical electrical sounding (VES) technique are widely used for a wide variety of exploration problems such as mineral explorations (e.g., Meiser, 1962; Yüngül, 1950, 1954), landslides (e.g., Bogoslovsky and Ogilvy, 1977), environmental problems (Bolève et al., 2011; Park et al., 2007), groundwater investigations (e.g., Göktürkler et al., 2008; Hamzah et al., 2007), geothermal explorations (e.g., Drahor and Berge, 2006; Özurlan et al., 2006), cave detection (e.g., Balkaya et al., 2012; Vichabian and Morgan, 2002) and archaeological prospection (e.g., Drahor, 2004; El-Qady et al., 1999).

Model parameters of SP and VES anomalies may be estimated by either local or global optimization methods that have both advantages and disadvantages relative to each other. Converging to the best-fitting solution using traditional gradient-based local-search optimization algorithms strongly depend on a good initial guess, while computationally expensive global-search algorithms using nature-inspired evolutionary algorithms (EAs) are not sensitive to the choice of the initial model (Başokur et al., 2007; Chunduru et al., 1997; Göktürkler, 2011). These algorithms as a sampler do not require a prior model. The only prior information for EAs is the search space, which can be enlarged especially in the presence of noisy data. Thus, they may be preferable to the local ones since prior information is not generally known (Fernández-Martínez et al., 2010b). Commonly used global optimization algorithms, which are based on direct search, include genetic algorithm (GA) (Holland, 1975), particle swarm optimization (PSO) (Kennedy and Eberhart, 1995) and simulated annealing (SA) (Kirkpatrick et al., 1983). GA (Abdelazeem and Gobashy, 2006), PSO (Monteiro Santos, 2010; Pekşen et al., 2011) and adaptive SA (ASA) (Tlas and Asfahani, 2008) were used for the interpretation of SP anomalies. Göktürkler and Balkaya (2012) performed a comparative study for these three algorithms to invert single SP anomalies caused by some polarized bodies with simple geometries. Additionally, GA (Balkaya et al., 2012; Fernández Alvarez et al., 2008; Jha et al., 2008), PSO (Fernández Martinez et al., 2010a; Shaw and Srivastava, 2007) and SA (Dittmer and Szymanski, 1995; Sen et al., 1993) were applied to invert VES data. Hybrid approaches combining local and global optimization algorithms were also used by researchers. For instance, Chunduru et al. (1997) used combined local conjugate gradient (CG) and global very fast SA (VFSA), and Başokur et al. (2007) used GA

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both as a stand-alone and as a hybrid approach together with iterative damped least-squares, namely Lamarckian inversion.

Differential evolution algorithm (DE), a class of EAs, was introduced by Storn and Price (1995) for solving a polynomial fitting problem. The algorithm is generally called as a very simple but very powerful population-based meta-heuristic algorithm (e.g., Qing, 2009, p. 41). DE is based on adding a weighted difference between two randomly chosen individuals from the population to a third one to find out new individuals in every generation (Storn and Price, 1997). It is similar to GA that is the most popular EA since DE uses the same genetic operators for optimization. But, DE uses these operators in a different order (mutation, crossover and selection) from GA (crossover, mutation and selection) in the reproduction cycle. In DE, mutation is carried out before crossover. DE includes ten different strategies: five different mutation implementations and two crossover operators (binomial and exponential) for each of them. The algorithm is generally characterized by the features of simplicity, effectiveness and robustness. Also, it is easy-to-use; and it requires few controlling parameters, and it has fast convergence characteristic (e.g., Noman et al., 2011; Storn and Price, 1997). Due to these advantages, it presents a wide range of implementation examples in different areas such as acoustics, biology, material science, mechanic, medical imaging, optic, mathematics, physics, seismology, etc. More details and examples about the implementation of DE to solve various problems are given in Qing (2009, pp. 41–51). Even though previous comprehensive studies over both common benchmark functions and real-world problems have shown that DE performs better in terms of convergence rate and robustness than the other EAs mentioned above, it has a very limited implementation in geophysical data inversion. DE has been used in geophysics for kinematic location of earthquake hypocenter (Růžek and Kvasnička, 2001); inversion of well-log data (Goswami et al., 2004); stochastic inversion of post-stack seismic data (Saraswat et al., 2010). Recently, Balkaya and Göktürkler (2012) and Li and Yin (2012) used DE for quantitative interpretation of self-potential data.

This study aimed to assess implementation of DE for inversion of geoelectrical data obtained by SP and VES studies. Three most frequently used mutation strategies with the binomial crossover type were used in the algorithm. Synthetically produced (i.e., noise-free and noisy) and two field data sets were considered in the SP implementation. Two known anomalies obtained over a Malachite mine (Jefferson County, Colorado) and over the ore bodies in the Neem-Ka-Thana, Rajasthan cooper belt (India) were used in the test studies. The solutions generated by DE were also compared with the results obtained by PSO that has been widely used to tackle geophysical inverse problems. Classical implementations of both algorithms were used in the comparison. The VES implementation includes interpretation of a three-layered synthetic resistivity curve and an example of a field data set from Gökçeada, Turkey. To clarify uncertainties in the solutions, model parameters from SP anomalies (electric dipole moment, polarization angle, depth, shape factor and origin of the anomaly) and VES curves (resistivity and thickness of the layers) were compared with the results from Metropolis–Hastings (M–H) sampling algorithm using SA without a cooling schedule. Comparison to the M–H algorithm indicates that DE performs a fast approximate posterior sampling for the case of low-dimensional inverse geophysical problems. As a result, DE can be considered as an effective algorithm by yielding compatible solutions with PSO in the inverse geoelectrical problems.

2. DE algorithm

DE algorithm (Price et al., 2005; Storn, 2008, pp. 1–31; Storn and Price, 1995) is one of the population-based global optimization algorithms having common sequence steps of an EA. Similar to GA; the algorithm has two stages including initialization and evolution. After randomly generating initial population by initialization, population evolves from one generation to the next through mutation, crossover and selection operations until the termination criteria are reached (Lin et al., 2011). One of the main differences between DE and GA is that the reproduction is carried out by a differential mutation before the crossover. Reproduction cycle to generate individuals for the next generation is carried out by basic operations of DE as described in the following sections.

2.1. Initialization

The algorithm begins by creating an initial population of target vectors consisting of parameters $x_{i,G} = (x^1_{i,G}, \ldots, x^D_{i,G})$, $i = (1, \ldots, Np)$ where Np is the population size, D is the number of parameters, G denotes the current generation, that is iteration in algorithm, and i is the index for individuals. The algorithm is initialized by a randomly created population within a predefined search space considering the upper and lower bounds of each parameter as follows:

$$x^j_{i,G} = x^j_l + rand(0,1)\cdot\left(x^j_u - x^j_l\right), \qquad j = 1, 2, \ldots, D \tag{1}$$

where j indicates parameters, l and u indicate lower and upper parameter bounds, respectively, and *rand()* represents a uniformly distributed random variable in the range of [0,1).

2.2. Mutation operation

This operation is performed after the initialization to create a mutant (donor) vector $v_{i,G} = (v^1_{i,G}, v^2_{i,G}, \ldots, v^D_{i,G})$ for each target vector. Table 1 shows the most common mutation schemes used in DE. Considering the classical approach in DE (the second strategy: DE/rand/1), three different vectors consisting of a base vector (x_{r_1}) and two difference vectors (x_{r_2} and x_{r_3}) are randomly chosen from the population. Mutation operation is then carried out by perturbing the base vector via a difference vector scaled by a weighting factor F (mutation constant).

In Table 1, $x_{best,G}$ is the best individual vector in the population at generation G, and indexes are random and mutually exclusive integers, and none of them corresponds to the base index i of current target vector. In order to describe different types of mutation schemes given in Table 1, a unique notation is generally used: *DE/x/y/z*, where x indicates how the base vector chosen (*rand*: vector is randomly selected and *best*: vector with the lowest objective function value), y indicates how many difference vector is added to it and z indicates what type of crossover method is chosen (i.e., binomial (*bin*) or exponential (*exp*)) (Price et al., 2005, p. 47).

2.3. Crossover operation

The trial vector is created by means of crossover operation once mutation operation has been terminated. It is realized between each pair of target vector ($x_{i,G}$) and its corresponding mutant vector ($v_{i,G}$), and can be simply formulated for the binomial uniform crossover that is widely used in the literature as shown below.

$$u^j_{i,G} = \begin{cases} v^j_{i,G} & \text{if}(rand(0,1) \le Cr \quad \text{or} \quad j = j_{rand}), \\ x^j_{i,G} & \text{otherwise}, \end{cases} \qquad j = 1, 2, \ldots, D \tag{2}$$

where Cr is a user-defined crossover probability in the range [0, 1], which controls the fraction of parameter values copied from the mutant vector, and j_{rand} is a randomly chosen integer in the range [1, D] to provide that the trial vector does not duplicate the target vector (Mandal et al., 2011; Price et al., 2005, p. 40).